CROSSFIRE

Uncoordinated network strategies for enhanced interference, mobility, radio resource, and energy saving management in LTE-Advanced networks

FP7 Contract Number: 317126

WP1 – Interference-Aware LTE-A Heterogeneous Wireless Networking

D1.2

Interference Modeling and Performance Assessment

<table>
<thead>
<tr>
<th>Contractual Date of Delivery:</th>
<th>February 2015</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actual Date of Delivery:</td>
<td>February 2015</td>
</tr>
<tr>
<td>Responsible Beneficiary:</td>
<td>CNRS</td>
</tr>
<tr>
<td>Contributing Beneficiaries:</td>
<td>VODAFONE, STEIN</td>
</tr>
<tr>
<td>Security:</td>
<td>Public</td>
</tr>
<tr>
<td>Nature:</td>
<td>Report</td>
</tr>
<tr>
<td>Version:</td>
<td>I</td>
</tr>
</tbody>
</table>

PROPRIETARY RIGHTS STATEMENT
This document contains information, which is proprietary to the CROSSFIRE Consortium. Neither this document nor the information contained herein shall be used, duplicated or communicated by any means to any third party, in whole or in parts, except with prior consent of the CROSSFIRE consortium.
1. Executive Summary

This deliverable is aimed at providing a comprehensive modeling and performance assessment of heterogeneous cellular networks for application to the LTE-A standard. This deliverable is divided into three main parts.

Part I encompasses Sections 1-4 and it is of methodological nature. It is aimed at developing new methodologies for modeling, analyzing and optimizing heterogeneous cellular networks. To this end, the mathematical tool of stochastic geometry is used. This part summarizes the research conducted by CNRS researchers, which was published in several journal papers. In particular, different methodologies are proposed for the analysis of the coverage probability, the average rate and the error performance of heterogeneous cellular networks, which use advanced transmission schemes, including multiple-antenna transmission, relay-aided communication and consider uplink and downlink scenarios. The proposed frameworks are shown to be highly accurate and suitable for system-level optimization. Based on them, several fundamental performance trends of heterogeneous cellular networks are identified.

Part II encompasses Section 5 and it is aimed at proposing a practical scheme for enhancing the throughput of heterogeneous cellular networks by introducing the idea of dual connectivity, i.e., the possibility of allowing different base stations to serve the mobile terminals in the downlink and uplink. This part is based on a patented technology from VODAFONE, which is called DUDE. In particular, it is shown that DUDE is capable of providing tenfold gains in the achievable throughput compared to the state-of-the-art LTE-A standard. It is considered, as a consequence, a suitable candidate for being considered in the next releases of the 5G standard.

Part III encompasses Section 6 and it is aimed at improving reliability, throughput and energy efficiency of the LTE-A standard by capitalizing on the principle of device-to-device communications. The proposed protocols are based on the concept of network coding and have been developed and implemented in the Steinwurf Kodo library and in the ns-3 simulator. The developed code and algorithms are available for free for academic/research use and can be downloaded from the Steinwurf website with a comprehensive tutorial guide. This is a fundamental step forward in order to stimulate and harmonize research in this field, which could eventually lead to the integration of network coding enhanced device-to-device technology into the next releases of the 5G standard.
Table of Contents

1. Executive Summary ............................................................................................................................................ 3

2. Performance Analysis of Downlink Heterogeneous Cellular Networks ............................................................... 7
   2.1 Stochastic geometry modeling and analysis of the error probability of single-tier SISO cellular networks: the Gil-Peleaz inversion approach ................................................................. 7
      2.1.1 Introduction .................................................................................................................................................. 7
      2.1.2 System Model .............................................................................................................................................. 9
      2.1.3 Characteristic Function of Interference ...................................................................................................... 10
      2.1.4 Error Performance ...................................................................................................................................... 12
      2.1.5 Numerical Results ...................................................................................................................................... 16
      2.1.6 Conclusion ................................................................................................................................................ 18
   2.2 Stochastic geometry modeling and analysis of the coverage probability and average rate of single-tier SISO cellular networks: the Gil-Peleaz inversion approach ........................................ 18
      2.2.1 Introduction ................................................................................................................................................ 19
      2.2.2 System Model and Problem Formulation .................................................................................................... 20
      2.2.3 Coverage Probability and Average Rate .................................................................................................... 21
      2.2.4 Gamma Distributed Per-Link Power Gains ............................................................................................... 23
      2.2.5 Numerical Results ...................................................................................................................................... 25
      2.2.6 Conclusion ................................................................................................................................................ 25
   2.3 Extension of the Gil-Peleaz inversion approach to single-tier MIMO cellular networks ..................................... 27
      2.3.1 Introduction ................................................................................................................................................ 27
      2.3.2 System Model .............................................................................................................................................. 27
      2.3.3 Characteristic Function of Interference ...................................................................................................... 29
      2.3.4 Error Performance ...................................................................................................................................... 30
      2.3.5 Numerical Results ...................................................................................................................................... 33
      2.3.6 Conclusion ................................................................................................................................................ 37
   2.4 Stochastic geometry modeling and analysis of the error probability of single-tier SISO cellular networks: the Equivalent-in-Distribution (EiD) based approach ........................................... 37
      2.4.1 Introduction ................................................................................................................................................ 37
      2.4.2 System model .............................................................................................................................................. 38
      2.4.3 Characteristic function of the interference ................................................................................................. 40
      2.4.4 The EiD-Based Approach .............................................................................................................................. 40
2.4.5 Performance Analysis ................................................................. 41
2.4.6 Numerical Results ...................................................................... 43

2.5 Extension of the Equivalent-in-Distribution (EiD) based approach to single-tier MIMO cellular networks ................................ 44
  2.5.1 Introduction ............................................................................. 44
  2.5.2 System model .......................................................................... 46
  2.5.3 Problem Formulation: Preliminaries ....................................... 47
  2.5.4 Main Results ........................................................................... 51
  2.5.5 Application to MIMO Cellular Networks ................................ 56

3. Performance Analysis of Uplink Heterogeneous Cellular Networks .... 69
  3.1 Introduction .................................................................................. 69
  3.2 Stochastic geometry modeling and analysis of the coverage probability and average rate of single-tier uplink cellular networks ......................................................... 69
    3.2.1 System Model and Problem Formulation ................................ 69
    3.2.2 Gil-Pelaez Based Mathematical Modeling .............................. 71
  3.3 Extension to multi-antenna uplink cellular networks with MRC receiver .... 72
    3.3.1 System Model ........................................................................ 72
    3.3.2 Gil-Pelaez Based Mathematical Modeling .............................. 74
  3.4 Extension to multi-tier heterogeneous uplink cellular networks .......... 75
    3.4.1 System Model and Problem Formulation ................................ 75
    3.4.2 Gil-Pelaez Based Mathematical Modeling .............................. 78
    3.4.3 Large Scale Receive Antenna System .................................... 79
  3.5 Numerical Results ........................................................................ 79

4. Performance Evaluation of Relay-Aided Downlink Cellular Networks .... 83
  4.1 Performance evaluation of cellular networks with fixed relays ........ 83
    4.1.1 Introduction .......................................................................... 83
    4.1.2 System Model ........................................................................ 85
    4.1.3 Network Interference Model .................................................. 87
    4.1.4 Problem Statement ................................................................ 88
    4.1.5 Diversity Order in Wireless Networks with Noise and Interference ................. 90
    4.1.6 End-to-end error probability of dual-hop cooperative relaying using a maximum ratio combining receiver ................................................................. 92
    4.1.7 End-to-end error probability of dual-hop cooperative relaying using a selection combining receiver ................................................................. 95
    4.1.8 Numerical and Simulation Results .......................................... 97
4.2 Performance evaluation of cellular networks with randomly distributed relays

4.2.1 Introduction ................................................................. 100
4.2.2 System Model ............................................................... 102
4.2.3 Association Policies ....................................................... 105
4.2.4 Simulation Results ......................................................... 106

5. Decoupled Uplink and Downlink Access in Future Cellular Networks (DUDe) ................................................................. 112

5.1 Introduction ........................................................................... 112
5.2 Motivation ............................................................................. 112
5.3 Toy example showing the concept .......................................... 114
5.4 Evaluation of the basic concept of DUDe ............................... 116
  5.4.1 System model .................................................................. 116
  5.4.2 Results ........................................................................... 119
5.5 Load and backhaul aware decoupled access (DUDe 2.0) .......... 124
  5.5.1 Motivation .................................................................... 124
  5.5.2 System model ................................................................. 125
  5.5.3 Cell association algorithm .............................................. 127
  5.5.4 Results ....................................................................... 128
5.6 RAN Architecture considerations for DUDe ......................... 135

6. Underlay Device-to-Device Communications for LTE-A Wireless Networks

6.1 Introduction ........................................................................... 138
6.2 RLNC transmission scenarios and schemes analysis ............... 139
  6.2.1 Network and System Model ............................................ 139
  6.2.2 Transmission Scenarios ................................................ 140
  6.2.3 Distribution Preliminaries .............................................. 141
  6.2.4 Single Field Schemes .................................................... 142
  6.2.5 Multiple Fields Schemes .............................................. 146
  6.2.6 Performance Metrics .................................................... 149
  6.2.7 Comparison Between Cooperative and Non-Cooperative Schemes ........................................................................... 153
6.3 RLNC Models for the ns-3 Simulator and the Steinwurf Kodo Library ......................................................... 155
6.4 Conclusions .......................................................................... 156

7. References ............................................................................... 157
2. Performance Analysis of Downlink Heterogeneous Cellular Networks

2.1 Stochastic geometry modeling and analysis of the error probability of single-tier SISO cellular networks: the Gil-Peleaz inversion approach

In this section, we introduce a mathematical framework for computing the average error probability of downlink cellular networks in the presence of other–cell interference, arbitrary fading, and thermal noise. A stochastic geometry based abstraction model for the locations of the Base Stations (BSs) is used, hence the BSs are modeled as points of a homogeneous spatial Poisson Point Process (PPP). The Mobile Terminal (MT) is assumed to be served by the BS that is closest to it. The technical contribution is twofold: 1) we provide an exact closed–form expression of the Characteristic Function (CF) of the aggregate other–cell interference at the MT, which takes into account the shortest distance based cell association mechanism; and 2) by relying on the Gil–Peleaz inversion theorem, we provide an exact closed–form expression of the Average Pairwise Error Probability (APEP), which accounts for fading and for the spatial distribution of the BSs. From the APEP, the Average Symbol Error Probability (ASEP) is obtained by using the Nearest Neighbor (NN) approximation, which is shown to provide accurate estimates. Finally, the mathematical framework is substantiated through extensive Monte Carlo simulations and insights on the achievable performance are discussed. More details including mathematical proofs, bounds and approximations can be found in [2.27].

2.1.1 Introduction

The mathematical modeling of cellular networks is usually conducted through abstraction models, which rely upon simplified spatial models for the locations of the Base Stations (BSs). Common approaches include the Wyner model, the single-cell interfering model and the hexagonal grid model [2.2][2.3]. However, these abstraction models are either inaccurate for many operating conditions or they still require extensive numerical computations. As a result, the analysis and design of cellular networks is often conducted by resorting to network simulations for selected scenarios, which represent specific arrangements of BSs.

To circumvent these problems, a new abstraction model for the mathematical analysis of cellular networks is emerging, which is referred to as Poisson Point Process (PPP)-based abstraction[2.2][2.3][2.29][2.37]. With the aid of this abstraction model, the locations of the BSs are modeled as points of a homogeneous PPP. Recent results have confirmed that the PPP-based abstraction model is capable of accurately reproducing the main structural characteristics of operational cellular networks [2.38]. The usefulness of the PPP-based
abstraction model originates from its analytical tractability and from the possibility of leveraging mathematical tools of applied probability, such as stochastic geometry, for mathematical performance analysis [2.47][2.49].

Owing to its mathematical tractability, the PPP-based abstraction model is now routinely used to the analysis and design of wireless networks in general and cellular networks in particular. Notable examples include[2.2][2.3], [2.4]-[2.7]. For a comprehensive literature survey, the interested reader is referred to [2.3][2.1]. More specifically, in [2.2] the coverage probability and the average rate of cellular networks are computed in closed-form for transmission over Rayleigh fading channels. In [2.4], the framework of [2.2] is extended to heterogeneous cellular networks, which are modeled as the superposition of many PPPs. In [2.5], the PPP-based abstraction model is exploited to study heterogeneous cellular networks with a biased cell association mechanism. In [2.6], the authors incorporate the load characteristics of the BSs into the mathematical framework, by using a conditionally thinning approach. In [2.7], a mathematical framework to the analysis of uplink of cellular networks is introduced. In [2.3], the authors introduce a framework for computing the average rate of heterogeneous cellular networks with biased cell association and for transmission over general fading channels.

The papers mentioned above focus their attention on the computation of the coverage/outage probability and of the average rate of a typical Mobile Terminal (MT). On the other hand, to the best of the authors knowledge, there are no mathematical frameworks to the analysis of the average error probability of PPP-based cellular networks. Indeed, the error probability and the outage probability in the presence of a Poisson field of interferers have been studied in the literature [2.8]-[2.11]. For example, in [2.10] a comprehensive framework is introduced for computing the error probability of a multi-antenna receiver in the presence of different models of network interference. By considering a similar interference model, the outage probability is studied in [2.9]. In [2.11], the effect of spatial interference correlation on the performance of maximum ratio combining is investigated. These frameworks, however, are not applicable to cellular networks, since the BS-to-MT cell association is neglected. The interferers are, in fact, assumed to be arbitrarily close to the typical MT, even closer than the serving BS. Thus, they are applicable to, e.g., ad hoc, cognitive and underlay device-to-device wireless networks, where the distance from the transmitter to the receiver is fixed and the interferers can be closer to the receiver than the intended transmitter.
Motivated by these considerations, in this section we introduce a mathematical framework for computing the error probability of SISO cellular networks, by explicitly taking into account the cell association mechanism based on the shortest BS-to-MT distance. The framework is applicable to cellular networks where the locations of the BSs are modeled according to a homogeneous PPP, the downlink channels experience independent and identically distributed fading. The mathematical approach is applicable to arbitrary fading distributions on both useful and interfering links. The proposed frameworks are useful for better understanding and for simplifying the analysis of cellular networks, since they do not require the explicit generation and simulation of the BSs locations, by using Monte Carlo simulations.

2.1.2 System Model

In cellular downlink, a probe MT is located at the origin of a bi-dimensional plane and BSs are modeled as points of a homogeneous PPP ($\Psi$) of density $\lambda$. The distance from the $i$th BS to the MT is denoted by $r_i$ for $i \in \Psi$. The MT is assumed to be tagged to the nearest BS. The serving BS is denoted by $\text{BS}_0$, and its distance from the MT is denoted by $r_0$, which is a RV with PDF $f_0(\xi) = 2\pi\lambda\xi \exp(-\pi\lambda\xi^2)$ [2.2]. The set of interfering BSs $i \in \Psi \backslash \{\text{BS}_0\}$ is still a homogeneous PPP [2.29] which is denoted by $\Psi^{(i)}$. $\Psi^{(i)}$ has density $p\lambda$, where $0 < p \leq 1$ is the activity factor denoting the probability that an interfering BS $i \in \Psi^{(i)}$ transmits in the same frequency band as $\text{BS}_0$. The setup with $p = 1$ corresponds to the full frequency reuse case [2.2], [2.3].

In the depicted downlink SISO cellular network model, the signal received at the MT is as follows:

$$y = \sqrt{\frac{E}{\lambda}} h_0 s_0 + \sqrt{E} \sum_{i \in \Psi^{(i)}} r_i^{-b} h_i s_i + n$$  \hspace{1cm} (2.1.1)

where $x$ is the useful signal transmitted by $\text{BS}_0$, $i_{agg}(r_0)$ is the aggregate other-cell interference and $n$ is the Additive White Gaussian Noise (AWGN) with noise power $N_0$. More specifically: $E$ is the BSs transmit-energy per transmission; $s_0$ is the information symbol emitted by $\text{BS}_0$, whereas $s_0 \in \mathbb{M}$ and the set $\mathbb{M}$ has $M$ modulated symbols denoted by $\mu \in \mathbb{C}$; $h_0$ is the channel matrix of the $\text{BS}_0$-to-MT link; $b > 1$ denotes the amplitude path-loss exponent. A similar notation is adopted for all the interfering channels of $i_{agg}(\cdot)$. The fading envelopes $|h_0|$ and $|h_i|$ for $i \in \Psi^{(i)}$ are assumed to be i.i.d. and to follow a generic distribution with
\[ \mathbb{E}\left\{ |h_i|^2 \right\} = \mathbb{E}\left\{ |h|^2 \right\} = \Omega. \] As an example, Gamma [2.24] and composite Gamma/Log-Normal [2.24] fading channels are explicitly analyzed.

At the MT, an interference-unaware Maximum-Likelihood (ML)-optimum demodulator is considered. It is assumed to have perfect Channel State Information (CSI) of the BS-to-MT link, while ignoring the other-cell interference. The decision metric \( \Lambda(\cdot) \) can be formulated as [2.24]:

\[
\Lambda(\tilde{s}_0) \propto r_0^{-2b} E[|\Delta_0|^2 |h_0|^2] + 2r_0^{-b} \sqrt{E}\text{Re}\left\{v(r_0)h_0^*\right\} 
\]

where \( \Delta_0 = s_0 - \tilde{s}_0 \), \( i_{agg}(r_0) = \sqrt{E} \sum_{i \in \Phi(0)} r_i^{-b} h_i s_i \) and \( v(r_0) = i_{agg}(r_0) + n \).

In this section, we provide a mathematical framework for computing the Average Symbol Error Probability (ASEP) of the demodulator in (2.1.2). The ASEP is defined as the symbol error probability averaged over the distribution of the fading channels and of the spatial locations of the BSs. The proposed approach for computing the ASEP consists of four steps:

1) first, the statistical distribution of \( i_{agg}(\cdot) \) is studied and a closed-form expression of its CF conditioned upon \( r_0 \) is derived in closed-form;

2) then, the Pairwise Error Probability (PEP) conditioned upon the fading envelope \( (h_0) \) and the transmission distance \( (r_0) \) of BSs is computed;

3) subsequently, the Average PEP (APEP) is obtained by removing the conditioning upon \( h_0 \) and \( r_0 \);

4) finally, the ASEP is computed from the APEP by using the NN approximation.

### 2.1.3 Characteristic Function of Interference

In this section, the CF of \( i_{agg}(\cdot) \) conditioned upon \( r_0 \) is computed in closed-form for arbitrary fading models.

**Theorem 2.1.1**

Let \( i_{agg}(r_0) = \sqrt{E} \sum_{i \in \Phi(0)} r_i^{-b} h_i s_i \). By conditioning upon \( r_0 \), its CF is as follows:
\[ \Phi_{agg}(\omega; r_0) = \Phi_{agg}(|\omega|; r_0) = \exp\left\{ -\lambda \pi r_0^2 T \left( |\omega|^2 r_0^2 E \right) \right\} \]  

(2.1.3)

where \( T(x) = pY(x) - p \) and:

\[ \Upsilon(x) = \frac{1}{M} \sum_{z=1}^{M} \sum_{q=0}^{+\infty} \left( -\frac{1}{4} \right)^q \frac{1}{q! (1-1/b)_q (1)_q} \left( x|\mu_x|^2 \right)^q E\{|h|^2| \}
\]

(2.1.4)

\[ = \frac{1}{M} \sum_{z=1}^{M} \sum_{|h|^2} E\left\{ F_2 \left( -\frac{1}{b};1,1-\frac{1}{b};-\frac{x}{4} |\mu_x|^2 |h|^2 \right) \right\} \]

where \( p F_q \left( a_1,\ldots,a_p; b_1,\ldots,b_q; \cdot \right) \) is the generalized hypergeometric function.

\[ \Box \]

Theorem 2.1.1 is general, as it is applicable to arbitrary fading distributions. However, the expectation over the fading square envelope \(|h|^2| \) of the generic interferer channel needs to be computed in (2.1.4). As an example, Gamma [2.24] and composite Gamma/Log-Normal [2.24] fading channels are explicitly analyzed.

**Gamma Channels**

Let \( g_i = |h|^2 \) follow a Gamma distribution with parameters \((m, \Omega)\), which we denote as \( g_i \sim \text{Gamma} \left( m, \Omega \right) \). Then, \( \Upsilon(\cdot) \) in (2.1.4) has closed-form expression as follows:

\[ \Upsilon(x) = \frac{1}{M} \sum_{z=1}^{M} \sum_{|h|^2} F_2 \left( -\frac{1}{b};m,1,1-\frac{1}{b};-\frac{1}{4} \frac{\Omega}{m} |\mu_x|^2 x \right) \]  

(2.1.5)

\[ \Box \]

**Gamma/LogNormal Channels**

Let \( g_i = |h|^2 \) follow a Gamma distribution by conditioning on its mean power, which follows a Log-Normal distribution. We denote this distribution as \( g_i \sim \text{Gamma/LogN} \left( m, \eta, \sigma^2 \right) \). Then, \( \Upsilon(\cdot) \) in (2.1.4) has closed-form expression as follows:

\[ \Upsilon(x) \approx \frac{1}{M} \sum_{z=1}^{M} \frac{1}{\sqrt{\pi}} \sum_{k=1}^{\sqrt{\pi} N_{\text{GHQ}}} w_k \bar{\tau}_k F_2 \left( -\frac{1}{b};m,1,1-\frac{1}{b};-\frac{1}{4} \frac{\bar{\tau}_k}{m} |\mu_x|^2 x \right) \]  

(2.1.6)

where \( \bar{\tau}_k = 10^{(5\bar{\tau}_k + 1)/10} \), and \( w_k \) and \( \bar{\tau}_k \) for \( k = 1,2,\ldots,N_{\text{GHQ}} \) are weights and abscissas of the Gauss-Hermite quadrature rule [2.35], respectively.

\[ \Box \]
Arbitrary Channels

Closed-form expressions of the CF of \( i_{\text{agg}}(\cdot) \) can be computed for a general class of fading distributions with the aid of the Meijer G-function. For a wide class of fading models, the PDF of \( g_i = |h_i|^2 \) can be formulated as

\[
f_{g_i}(\xi) = C \xi^{\alpha} G_{p,q}^{a,b} \left( \begin{pmatrix} a_p \\ b_q \end{pmatrix} \right) \left( \begin{pmatrix} x \\ 4 \end{pmatrix} \right)\]

\[
D \xi^{\beta} G_{p,q}^{a,b} \left( \begin{pmatrix} a_p \\ b_q \end{pmatrix} \right) \left( \begin{pmatrix} x \\ 4 \end{pmatrix} \right)
\]

where

\[
G_{p,q}^{a,b} \left( \begin{pmatrix} a_p \\ b_q \end{pmatrix} \right)
\]

is the Meijer G-function. With the aid of [2.22] the expectation in (2.1.4) and can be re-written as follows:

\[
E_{c_i} \left\{ \text{IF}_2 \left( -\frac{1}{b} ; 1,1 ; -\frac{1}{b} \frac{x}{4} g_i \right) \right\} = -\frac{C}{b} \int_0^{\infty} \xi G_{1,3}^{1,1} \left( \begin{pmatrix} x/4 \\ 0 \end{pmatrix} \right) G_{p,q}^{a,b} \left( \begin{pmatrix} a_p \\ b_q \end{pmatrix} \right) d\xi
\]

which can be solved in closed-form with the aid of the Mellin-Barnes theorem in [2.22].

\[\square\]

2.1.4 Error Performance

APEP

By definition, the APEP is the probability that the actual transmitted symbol \( s_0 \) is decoded as \( \hat{s}_0 = \bar{s}_0 \neq s_0 \), by assuming that \( s_0 \) and \( \bar{s}_0 \) are the only two symbols of the constellation diagram. From (2.1.2), this occurs when \( \Lambda(\bar{s}_0 = \bar{s}_0) < \Lambda(\bar{s}_0 = s_0) \). Since \( \Lambda(\bar{s}_0 = s_0) = 0 \) from (2.1.2), the APEP reduces to the computation of

\[
\text{APEP}(\Delta_0) = \Pr \{ \Lambda(\bar{s}_0 = \bar{s}_0) < 0 \}
\]

We start by computing the APEP conditioned upon \( |h_0| \) and \( r_0 \), which is denoted by \( \text{PEP}(\Delta_0; |h_0|, r_0) \). Hence, the APEP is

\[
\text{APEP}(\Delta_0) = \text{PEP}(\Delta_0; |h_0|, r_0)
\]

From (2.1.2), the APEP can be formulated as summarized in the following theorem.

Theorem 2.1.2

Let the demodulator in (2.1.5). Let \( \text{SNR} = E/No \). The APEP can be formulated as

\[
\text{APEP}(\Delta_0) = \Pr \{ \Lambda(\bar{s}_0) < 0 \} = \text{APEP}_n(\Delta_0) + \text{APEP}_n(\Delta_0)
\]

where:
Theorem 2.1.2 provides an exact mathematical formulation of the APEP, which is applicable to arbitrary distributions of the fading envelope $|h_0|$. The expectations in (2.1.9) can be computed in closed-form for various fading models. As examples, closed-form expressions of $Q_N(\cdot;\cdot)$ and $Q_{NI}(\cdot;\cdot)$ are provided for Gamma and Gamma/Log-Normal channels.

**Gamma Channels**

Let $g_0 = |h_0|^2 \sim \text{Gamma}(m, \Omega)$. Let $K_1 \geq 0$ and $K_2 \geq 0$ two non-negative constants. Let the expectations:

$$I_1(K_1) = E_{g_0}\left\{\sqrt{g_0} \exp\{-K_1g_0\}\right\} = \frac{m}{\Omega^m \Gamma(m)} \Gamma\left(m + \frac{1}{2}\right) \left(K_1 + \frac{m}{\Omega}\right)^{-(m+1/2)}$$

$$I_2(K_2) = E_{g_0}\left\{\sin(K_2\sqrt{g_0})\right\} = \frac{\sqrt{\Omega}}{m} \Gamma\left(m + \frac{1}{2}\right) \frac{K_2\mathcal{F}_1\left(m + \frac{1}{2}; \frac{3}{2}; \frac{\Omega K_2^2}{4m}\right)}{\Gamma(m)}$$

Then, $Q_N\left(x;|\Delta_0|, \text{SNR}, b\right) = I_1\left((1/4)\text{SNR}|\Delta_0|^2 x^{-2b}\right)$ and $Q_{NI}\left(x;|\Delta_0|\right) = I_2\left((1/2)|\Delta_0| x^{1/2}\right)$.

**Gamma/LogNormal Channels**

Let $g_0 = |h_0|^2 \sim \text{Gamma/LogN}(m, \eta, \sigma^2)$. Let $K_1 \geq 0$ and $K_2 \geq 0$ two non-negative constants. Let the expectations:
\[
I_1(K_1) = E_{\hat{s}_0} \left\{ \sqrt{g_0 \exp \{ -K_1 g_0 \}} \right\}
\approx \frac{1}{\sqrt{\pi}} \frac{m^m \Gamma \left( m + \frac{1}{2} \right)}{\Gamma(m)} \sum_{k=1}^{N_{\text{cas}}} \hat{\tau}_k^{-m} \left( K_1 + \frac{m}{\hat{\tau}_k} \right)^{-\left( m+\frac{1}{2} \right)}
\]
\[
I_2(K_2) = E_{\hat{s}_0} \left\{ \sin \left( K_2 \sqrt{g_0} \right) \right\}
\approx \frac{\Gamma \left( m + \frac{1}{2} \right)}{\Gamma(m)} K_2 \frac{1}{\sqrt{\pi}} \sum_{k=1}^{N_{\text{cas}}} \hat{\tau}_k \left( m + \frac{1}{2} \right) \left( \frac{3}{2} \hat{\tau}_k^2 - \frac{1}{4m} \right)
\]

Then, 
\[
Q_N(x;|\Delta_0|,\text{SNR},b) = I_1 \left( \left( 1/4 \right) \text{SNR} |\Delta_0|^2 x^{-2b} \right) \quad \text{and} \quad Q_{NI}(x;|\Delta_0|) = I_2 \left( (1/2) |\Delta_0| x^{1/2} \right).
\]

\textbf{Asymptotic Analysis}

We then provide simplified mathematical frameworks in two limiting operating conditions: 1) noise-limited, i.e., \( \text{SNR} \to 0 \), and 2) interference-limited, i.e., \( \text{SNR} \to +\infty \), cellular networks, where \( \text{SNR} = E / N_0 \) denotes the Signal-to-Noise-Ratio (SNR). As discussed in[2.3], typical cellular networks operate in the interference-limited regime.

\textbf{Corollary 2.1.1}

Let a noise-limited cellular network. Then,

\[
\lim_{\text{SNR} \to 0} \{ \text{APEP}(|\Delta_0|) \} = \text{APEP}_N^0 \left( |\Delta_0| \right) = \text{APEP}_N \left( |\Delta_0| \right).
\]

Let an interference-limited network.

Then,

\[
\lim_{\text{SNR} \to +\infty} \{ \text{APEP}(|\Delta_0|) \} = \text{APEP}_{NI}^{(o)} \left( |\Delta_0| \right):
\]

\[
\text{APEP}_{NI}^{(o)} \left( |\Delta_0| \right) = \lim_{\text{SNR} \to +\infty} \{ \text{APEP}_{NI}(|\Delta_0|) \}
= (2\pi)^{-1} \int_0^{+\infty} (1/x) T(x) (1+T(x))^{-1} Q_{NI} \left( x; |\Delta_0|, b \right) dx
\]

\textbf{ASEP}

From the APEP inTheorem 2.1.1, the ASEP can be obtained by using the NN approximation [2.30]. In this section, we propose the NN approximation for computing the ASEP from the APEP because of its simplicity and accuracy. The advantage is, in fact, that the ASEP is obtained by computing a single APEP for every symbol \( \mu_j \) for \( j = 1, 2, \ldots, M \) of the constellation diagram. By assuming equiprobable transmitted symbols, the NN approximation of the ASEP can be formulated as follows:
ASEP \approx \left(1/\mathcal{M}\right) \sum_{x=1}^{\mathcal{M}} N_{\Delta_{\min}}^{(x)} \text{APEP}\left(\left|\Delta_{\min}^{(x)}\right|\right)

(2.1.13)

where: i) \(\left|\Delta_{\min}^{(x)}\right| = \min_{x \neq \tilde{x}, \mu_{\tilde{x}} \in \mathcal{M}} \left|\left|\mu_{\tilde{x}} - \mu_{x}\right|\right|\) is the minimum Euclidean distance among all pairs \((\mu_{\tilde{x}}, \mu_{x})\) of the constellation diagram for \(\mu_{\tilde{x}} \in \mathcal{M}\) and ii) \(N_{\Delta_{\min}}^{(x)}\) is the number of nearest neighbors of \(\mu_{x}\), i.e., the number of points of the constellation diagram whose Euclidean distance from \(\mu_{x}\) is equal to \(\left|\Delta_{\min}^{(x)}\right|\).

For some constellation diagrams, we may have \(\left|\Delta_{\min}^{(x)}\right| = \left|\Delta_{\min}\right|\), which is independent of \(\mu_{x}\).

In this case, (2.1.13) reduces to \(\text{ASEP} \approx N_{\Delta_{\min}}^{(\text{avg})} \text{APEP}\left(\left|\Delta_{\min}\right|\right)\), where \(N_{\Delta_{\min}}^{(\text{avg})} = \left(1/\mathcal{M}\right) \sum_{x=1}^{\mathcal{M}} N_{\Delta_{\min}}^{(x)}\) is the average number of nearest neighbors of the constellation diagram. For example, \(\left|\Delta_{\min}\right| = 2 \sin\left(\pi/\mathcal{M}\right)\) with \(N_{\Delta_{\min}}^{(\text{avg})} = 1\) if \(\mathcal{M} = 2\) and \(N_{\Delta_{\min}}^{(\text{avg})} = 2\) if \(\mathcal{M} > 2\) for PSK modulation. If the standard square QAM with \(\mathcal{M} = 16\) is considered, we have \(\left|\Delta_{\min}\right| = 2/\sqrt{10}\) and \(N_{\Delta_{\min}}^{(\text{avg})} = 3\). Similar results apply for different values of \(\mathcal{M}\).

**Trends and Insights**

Let us consider a SISO cellular network operating in the noise-limited regime. It follows that \(\text{APEP}\left(\left|\Delta_{0}\right|\right) \approx \text{APEP}_{N}\left(\left|\Delta_{0}\right|\right)\). Thus, the following conclusions can be drawn.

- The APEP decreases by increasing the SNR.
- The diversity order, \(D\), depends on \(Q_{N}^{(x)} = Q_{N}^{(x)}\) as \(\text{SNR} \rightarrow \infty\).
- The APEP decreases by increasing the BSs density.
- The APEP increases by increasing the shadowing standard deviation.
- The APEP increases by increasing the path-loss exponent.

Let us consider a SISO cellular network operating in the interference-limited regime. It follows that \(\text{APEP}\left(\left|\Delta_{0}\right|\right) \approx \text{APEP}_{N}^{(x)}\left(\left|\Delta_{0}\right|\right)\). Thus, the following conclusions can be drawn.

- The APEP is independent of the SNR.
- The APEP is independent of the BSs density.
- We expect that the APEP decreases by increasing \(m\).
We expect that the APEP degrades by increasing $\sigma$. However, it is very difficult to study the monotonicity of the hypergeometric functions as a function of their many parameters.

- The APEP decreases by increasing the path-loss exponent.
- The APEP decreases by decreasing the activity factor.

All the trends inferred from mathematical frameworks can be supported by Monte Carlo Simulations, which are shown in the next section.

### 2.1.5 Numerical Results

We validate in this section the mathematical frameworks against Monte Carlo simulations, which are obtained by using the procedure described in [2.3], and study the impact of different parameters.

Before that, we first validate the PPP-based abstraction model by comparing it with modeling the BS locations via grid-based model. A similar comparison is available in [2.2] for the coverage probability. We provide in this section the comparison for ASEP. Fig. 2.1.1 shows that the PPP-based abstraction model (“solid lines”) provides a worst estimate of the error probability compared to the grid-based abstraction model (“dash curves”), since interfering BSs may be arbitrarily close to each other.

On the other hand, the PPP-based abstraction model provides tractability and performance trends as a function of system parameters can be inferred from the resulting mathematical frameworks. We observe a good accuracy of the obtained frameworks (depicted in curves with colored markers) compared with the Monte Carlo simulations (depicted in “black” curves).

Figs. 2.1.1 – 2.1.3 all support the inferred trends and insights from mathematical frameworks. More specially, Fig. 2.1.1 confirms that ASEP decreases by decreasing the activity factor. Fig. 2.1.2 shows that by increasing the path-loss exponent, ASEP decreases in the interference-limited regime while increases in the noise-limited regime. Fig. 2.3.3 confirms that the BS density affects the ASEP only in the noise-limited regime.
Figure 2.1.1 ASEP of a SISO system against the transmit SNR. Setup: 16QAM, Gamma/Log-Normal channel model with $\sigma = 6\text{dB}$, $m = 2$, $\lambda = 10^{-5}$ and $b = 3$.

Figure 2.1.2 ASEP of a $N_t \times N_r = 1 \times 1$ system against the transmit SNR $E/N_0$. Setup: (a) QAM with $M = 16$, Gamma/Log-Normal channel model with $\sigma = 6\text{dB}$, $b = 3$, $\lambda = 10^{-5}$ and $p = 10^{-3}$. (b) QAM with $M = 16$, Gamma/Log-Normal channel model with $m = 2$, $b = 3$, $\lambda = 10^{-5}$ and $p = 10^{-3}$.
2.1.6 Conclusion

In this section, we have proposed a new mathematical framework for computing the average error probability of downlink cellular networks by relying on a PPP–based abstraction model for the locations of the BSs. A new closed–form expression of the CF of the aggregate other–cell interference has been proposed, and an easy–to–compute integral expression of the ASEP has been provided. The mathematical framework is applicable for general fading models and has been substantiated with the aid of Monte Carlo simulations and a good accuracy has been observed. From the mathematical framework, various performance trends have been identified, which have been confirmed by Monte Carlo simulations.

2.2 Stochastic geometry modeling and analysis of the coverage probability and average rate of single-tier SISO cellular networks: the Gil-Peleaz inversion approach

In this section, we introduce new mathematical frameworks to the computation of coverage probability and average rate of cellular networks, by relying on a stochastic geometry
abstraction modeling approach. With the aid of the Gil-Pelaez inversion formula, we prove that coverage and rate can be compactly formulated as a twofold integral for arbitrary per-link power gains. In the interference-limited regime, single-integral expressions are obtained. As a case study, Gamma-distributed per-link power gains are investigated further, and approximated closed-form expressions for coverage and rate in the interference-limited regime are obtained, which shed light on the impact of channel parameters and physical-layer transmission schemes. More details including mathematical proofs can be found in [2.28].

2.2.1 Introduction

Recently, different techniques to the mathematical modeling and performance evaluation of cellular networks based on stochastic geometry have been reported[2.1]. To the best of the authors’ knowledge, several techniques are commonly used to the computation of important performance metrics, which include coverage probability, average rate and error probability. They offer a different trade-off in terms of modeling accuracy, mathematical tractability, numerical complexity, etc. [2.1]. These techniques can be classified as based on:

1) the Rayleigh fading assumption [2.1], [2.2];
2) the dominant or nearest interferers approximation [2.1];
3) approximations of the distribution of the other-cell interference [2.1];
4) the Plancherel-Parseval theorem that is applicable to arbitrary fading for the desired link [2.1], [2.2];
5) numerically inverting the Moment Generating Function (MGF) of the other-cell interference;
6) MGF-based equivalent representations of the performance metrics of interest[2.3];
7) equivalent in distribution representations of the other-cell interference[2.12]; and 8) the direct computation of spatial averages without using the MGF [2.13].

In this section, we introduce another technique to the computation of coverage and rate of cellular networks. The proposed approach is based on the Gil-Pelaez inversion formula [2.14]. The application of the Gil-Pelaez theorem to the analysis of wireless networks in the presence of interference is not new and various papers are available, e.g., [2.15], [2.16]. These mathematical frameworks, however, are not based on a stochastic geometry abstraction modeling. In this context, to the best of the author’s knowledge, the Gil-Pelaez theorem has been employed only in [2.17] and [2.18], where the error probability of cognitive radio and cellular networks is investigated, respectively. In this section, on the other hand, we are interested in the analysis of coverage and rate of cellular networks, which lead to a different and novel mathematical formulation. More specifically, we provide novel two-fold
integral expressions for coverage and rate, which have a compact mathematical formulation and are applicable to arbitrary per-link power gains and path-loss exponents. In the interference-limited regime, exact single-integral expressions formulated in terms of generalized hypergeometric functions are provided.

As a case study, we focus our attention on fading channels and transmission schemes whose equivalent per-link power gains follow a Gamma distribution with arbitrary parameters. In this scenario, we provide approximated closed-form expressions for coverage and rate, whose accuracy is assessed with the aid of Monte Carlo simulations. The rationale of this choice originates from [2.19] and [2.20], where it is shown that the per-link power gains of a large class of multiple-antenna transmission schemes for transmission over Rayleigh fading channels can be approximated by a Gamma distribution with adequate parameters. In [2.19], the impact of the parameters of the Gamma distribution is investigated by relying on approximated expressions of the other-cell interference obtained through moment-matching methods. In [2.20], the same problem is solved with the aid of stochastic ordering. In [2.1], it is shown that, in general, the analysis of multiple-antenna transmission schemes requires the computation of the derivatives of the MGF of the other-cell interference. In this section, approximated but simple closed-form expressions for coverage and rate are provided, which provide insight on the achievable performance of cellular networks as a function of the parameters of the per-link power gains, e.g., the multiple-antenna transmission scheme if Rayleigh fading is assumed.

2.2.2 System Model and Problem Formulation

Similar to the case study of the error performance in Section 2.1, we assume a probe MT is located at the origin of bi-dimensional plane and the BSs are modeled as points of a homogeneous PPP (\( \Psi \)) of density \( \lambda \). The distance from the \( i \) th BS to the MT is denoted by \( r_i \) for \( i \in \Psi \). The MT is assumed to be tagged to the nearest BS. The serving BS is denoted by \( \text{BS}_0 \), and its distance from the MT is denoted by \( r_0 \), which is a RV with PDF \( f_{r_0}(\xi) = 2\pi \lambda \xi \exp(-\pi \lambda \xi^2) \). The set of interfering BSs \( i \in \Psi \setminus \{\text{BS}_0\} \) is still a homogeneous PPP which is denoted by \( \Psi^{(\text{agg})} \). \( \Psi^{(\text{agg})} \) has density \( \lambda \).

The SINR of this downlink cellular network can be formulated as follows:

\[
\text{SINR} = \frac{P\gamma_0 r_0^{-\alpha}}{\sigma_N^2 + PI_{\text{agg}}(r_0)}; \quad I_{\text{agg}}(r_0) = \sum_{i \in \Psi^{(\text{agg})}} \gamma_i r_i^{-\alpha}
\]  
(2.2.1)
where $P$ is the BSs transmit-energy per transmission; $\sigma_n^2$ is the noise power, $\alpha > 2$ denotes the path-loss exponent. $I_{agg}(\cdot)$ is the aggregate other-cell interference, $\gamma_0$ and $\gamma_i$ for $i \in \Psi^{(\omega)}$ are the per-link power gains of intended and interfering links, which have an arbitrary distribution that usually depends on fading channel and transmission scheme [2.19][2.20].

The coverage probability ($P_{cov}$) and average rate ($R$) are studied. They can be formulated as follows:

$$P_{cov}(T) = \Pr \{\text{SINR} \geq T\}$$  \hspace{1cm} (2.2.2)$$

$$R = \mathbb{E} \{\ln(1 + \text{SINR})\} = \int_0^\infty P_{cov}(\exp(t) - 1)\,dt = -\int_0^\infty \ln(1 + y)P_{cov}^{(1)}(y)\,dy$$  \hspace{1cm} (2.2.3)$$

where $T > 0$ is a reliability threshold, (a) follows from [2.1] and (b) follows by applying integration by parts, since $P_{cov}(T \to 0) = 1$ and $P_{cov}(T \to \infty) = 0$.

### 2.2.3 Coverage Probability and Average Rate

New mathematical frameworks to the computation of (2.2.1) and (2.2.2) are provided, by assuming that $\gamma_0$ and $\gamma_i$ for $i \in \Psi^{(\omega)}$ have an arbitrary distribution. For generality, and according to, e.g., [2.19][2.20], the distributions of $\gamma_0$ and $\gamma_i$ are different and independent.

The interferers power gains $\gamma_i$ for $i \in \Psi^{(\omega)}$ on the other hand, are assumed to be independent but identically distributed.

**Theorem 2.2.1**

$P_{cov}$ in (2.2.1) can be formulated as:

$$P_{cov}(T) = \frac{1}{2} - 2\lambda \int_0^\infty \text{Im} \left\{ M_{\gamma_0} \left( j \frac{x}{T} \right) F_{\gamma_i}(x) \right\} \frac{dx}{x}$$  \hspace{1cm} (2.2.4)$$

where $M_s(t) = \mathbb{E} \{\exp(-sx)\}$ is the MGF of RV $x$ and the following functions are introduced:

$$F_{\gamma_i}(x) = \int_0^\infty y \exp \left( jy^\alpha x + \frac{\sigma_n^2}{P} \right) \exp(-\pi\lambda y^2\gamma_i(jx))\,dy$$  \hspace{1cm} (2.2.5)$$

$$\gamma_i(z) = \mathbb{E}_y \left\{ \frac{2}{\alpha} \right\}$$  \hspace{1cm} (2.2.6)$$

where $\text{}_pF_q\left(a_1, \ldots, a_p; b_1, \ldots, b_q; \cdot \right)$ is the generalized hypergeometric function.
Proof: See [2.28]. □

Corollary 2.2.1

\[ P_{\text{cov}} \text{ in (2.2.1) when } \sigma_N^2 = 0 \text{ can be formulated as:} \]

\[ P_{\text{cov}}^{[\alpha]}(T) = \frac{1}{2} - \frac{1}{\pi} \int_0^\infty \text{Im} \left\{ \frac{M_{\sigma_\alpha} \left( \frac{jx}{T} \right)}{Y_j(jx)} \right\} \frac{dx}{x} \quad (2.2.7) \]

Proof: See [2.28]. □

The computation of (2.2.4) and (2.2.7) requires a closed-form expression of the expectation in (6). Remark 2.2.1 provides a general approach to solve this problem.

Remark 2.2.1

Let \( E_{\gamma_k} \{ \gamma_k \} = \bar{y}^k \left( \begin{pmatrix} \mathbf{p} \\ \mathbf{q} \end{pmatrix} \right)_k \) for \( k = 1, 2, ... \), where \( \mathbf{p} = (p_1, p_2, ..., p_N) \) and \( \mathbf{q} = (q_1, q_2, ..., q_M) \) are vectors with N and M real-valued entries, and \( (\mathbf{p})_k \) and \( (\mathbf{q})_k \) are short-hands for \( \sum_{k=0}^\infty \frac{(-2/\alpha)_k}{(1-2/\alpha)_k} (\mathbf{p})_k (\mathbf{q})_k \) respectively where \( (\cdot)_k \) denotes the Pochhammer symbol. Then the following equalities hold:

\[ Y_j(z) = \sum_{k=0}^\infty \frac{(-2/\alpha)_k}{(1-2/\alpha)_k} (\mathbf{p})_k (\mathbf{q})_k \frac{(z\bar{y})^k}{k!} = \frac{2}{\alpha} \mathbf{F}_{M+1} \left( 1-\frac{2}{\alpha} \mathbf{q}; z\bar{y} \right) \quad (2.2.8) \]

It follows from the series representation of the hypergeometric function[2.21]. The formulation holds for a large number of fading distributions and transmission schemes. A general class of distributions is available in[2.23], whose moments can be computed using [2.22].

Theorem 2.2.2

\( R \) in (2.2.3) can be formulated as follows:

\[ R = 2\lambda \int_0^\infty \text{Im} \left\{ jF_0 \left( jx \right) F_M \left( x \right) \right\} \frac{dx}{x} \quad (2.2.9) \]

Where the following functions are introduced:

\[ F_0 \left( jx \right) = \int_0^\infty \frac{\ln(1+y)}{y^2} M_{\gamma_0} \left( \frac{z}{y} \right) dy \quad (2.2.10) \]

Proof: See [2.28]. □

Corollary 2.2.2

\( R \) in (2.2.3) when \( \sigma_N^2 = 0 \) can be formulated as follows:
\[ R^{[\nu]} = -\frac{1}{\pi} \int_0^\infty \text{Im} \left\{ \frac{F_0(jx)}{Y_I(jx)} \right\} \frac{dx}{x} \]  \hspace{1cm} (2.2.11)

**Proof:** See [2.28]. □

**Remark 2.2.2**

The computation of (2.2.10) and (2.2.11) requires closed-form expressions for the first derivative \( M^{(1)}_\nu (.) \). Luckily, they are available for many fading distributions. A summary can be found in [2.23].

### 2.2.4 Gamma Distributed Per-Link Power Gains

In this section, we focus our attention on the case study where the power gains of intended and interfering links follow a Gamma distribution with arbitrary parameters, i.e.,

\[
\gamma_0 \sim \text{Gamma}\left(m_0, \Omega_0\right) \quad \text{and} \quad \gamma_I \sim \text{Gamma}\left(m_I, \Omega_I\right).
\]

This case study is meaningful because it finds application to the analysis of cellular networks for propagation over Rayleigh fading, which rely on multiple antenna transmission schemes at the physical layer. The reader is referred to [2.19][2.20] for further details. In order to get insight on the impact of the multiple-antenna transmission scheme, the authors of [2.19][2.20] resort to approximated representations of the aggregate other-cell interference and to stochastic ordering analysis. In [2.1], it is shown that an accurate analysis of this scenario would require the computation of the higher-order derivatives of the MGF of the aggregate other-cell interference. Unlike these papers, with the aid of the mathematical formulations in Corollary 1 and Corollary 2, we propose approximated but closed-form expressions for coverage and rate. The obtained mathematical expressions are shown to provide relevant information on the impact of system parameters, which may offer a simple approach for comparing various multiple-antenna transmission schemes at the physical layer [2.19][2.20]. In Section V, we show that the performance trends obtained from the proposed mathematical frameworks are confirmed with the aid of Monte Carlo simulations.

By assuming \( \gamma_0 \sim \text{Gamma}\left(m_0, \Omega_0\right) \) and \( \gamma_I \sim \text{Gamma}\left(m_I, \Omega_I\right) \), Corollary 1 and Corollary 2 can be simplified as follows:

\[
P^{[\nu]}_{\text{cov}}(T) = \frac{1}{2} - \frac{1}{\pi} \int_0^\infty \text{Im} \left\{ \frac{(1 + j\kappa_0 x)^{-m_0}}{F_1\left(-\frac{2}{\alpha}, m_I, 1 - \frac{2}{\alpha}; j\kappa_I x\right)} \right\} \frac{dx}{x} \]  \hspace{1cm} (2.2.12)
\[ R^{[\kappa]} = \frac{1}{\pi \Gamma(m_0)} \int_0^\infty \text{Im} \left[ G_{3,3}^{2,3} \left( j \kappa_0 x; \begin{array}{c|cc} 1 & 1 & 1-m_0 \\ \hline 1 & 1 & 0 \end{array}; \frac{2}{\alpha}, m_0 ; 1-\frac{2}{\alpha} ; j \kappa_i x \right) \right] dx \]  

(2.2.13)

where \( \kappa_0 = \Omega_0 / m_0, \tilde{\kappa}_0 = \kappa_0 / T \) and \( \kappa_i = \Omega_i / m_i \). \( G_{p,q}^{\alpha,\beta} \left( \begin{array}{c} a_p \\ b_q \end{array} \right) \) is the Meijer G-function.

**Proof:** See [2.28]. □

Approximated closed-form expressions of (2.2.12) and (2.2.13) are provided in Proposition 1 and Proposition 2, respectively.

**Proposition 2.2.1**

\[ P_{\text{cov}}^{[\kappa]} \text{ in (2.2.12) can be approximated as:} \]

\[ P_{\text{cov}}^{[\kappa]}(T) = 1 - \left( 1 + \frac{1}{2} \frac{\Omega_0}{\Omega_i} (\alpha - 2) \frac{1}{T} \right)^{-m_0} \]  

(2.2.14)

**Proof:** See [2.28]. □

**Remark 2.2.3:**

By direct inspection of (2.2.14), the coverage probability has the following performance trends: i) it increases as \( \eta = \Omega_0 (\alpha - 2) / (T \Omega_i) \) increases; ii) it increases as \( m_0 \) increases; and iii) it is independent of \( m_i \).

**Remark 2.2.4:**

Let \( K_{\text{cov}}^{[\kappa]} = (\Omega_0 / \Omega_i) (1 / (m_0 T)) \). The accuracy of the approximation in (2.2.14) increases as \( K_{\text{cov}}^{[\kappa]} \) increases.

**Proof:** See [2.28]. □

**Proposition 2.2.2**

\[ R^{[\kappa]} \text{ in (2.2.13) can be approximated as:} \]

\[ R^{[\kappa]} = \frac{1}{\Gamma(m_0)} \cdot G_{3,3}^{2,3} \left( \begin{array}{c|cc} 1 & 1 & 1-m_0 \\ \hline 1 & 1 & 0 \end{array}; \frac{1}{2}, m_0 ; \frac{1}{2} \frac{\Omega_0}{\Omega_i} ((\alpha - 2))^{-1} \right) \quad (2.2.15) \]

**Proof:** See [2.28]. □
Remark 2.2.5:
By plotting (2.2.15) as a function of $\tilde{\eta} = (\Omega_0 / \Omega_f) (\alpha^2)$, the same trends as in coverage probability (Remark 3) hold.

Remark 2.2.6:
Let $K_{\text{Rate}} = (\Omega_0 / \Omega_f) (1 / m_0)$. The accuracy of the approximation in (2.2.15) increases as $K_{\text{Rate}}$ increases.

2.2.5 Numerical Results
In Figs. 2.2.1 and 2.2.2, numerical examples are shown to substantiate the proposed mathematical frameworks (“solid lines”), approximations (“markers”) against Monte Carlo simulations (“black dots”), which are obtained as described in [2.3]. For simplicity, the interference-limited regime is analyzed, i.e., $\sigma^2 = 0$ and the per-link power gains are Gamma distributed. The illustrations confirm that (2.2.12) and (2.2.13) are exact. They also show that (2.2.14) and (2.2.15) are fairly accurate. In particular, the expected accuracy discussed in Remark 2.2.4 and Remark 2.2.6 is confirmed. More importantly, (2.2.14) and (2.2.15) well reproduce the behavior of coverage and rate as a function of the system parameters, as discussed in Remark 2.2.3 and Remark 2.2.5. This is, in fact, the main usefulness of these approximations. Furthermore, the figures show that the accuracy of (2.2.14) and (2.2.15) increases as $\alpha$ decreases. This is an important outcome, since Monte Carlo simulations are either less accurate or require more simulation time for small values.

2.2.6 Conclusion
In section 2.2, new mathematical expressions for coverage and rate of cellular networks are provided with the aid of the Gil-Pelaez inversion formula. The frameworks are shown to be general enough for the analysis of different fading channels and transmission schemes. Furthermore, closed-form approximated expressions are proposed when the per-link power gains are distributed according to a Gamma distribution.
Figure. 2.2.1  $P_{\text{cov}}^{[\alpha]}$ as a function of $T$ and $K = (\Omega_0 / \Omega_I)(1/m_0)$

Figure. 2.2.2  $R^{[\alpha]}$ as a function of $\alpha$ and $K = (\Omega_0 / \Omega_I)(1/m_0)$
2.3 Extension of the Gil-Peleaz inversion approach to single-tier MIMO cellular networks

In this section, a mathematical framework to evaluate the error performance of downlink Multiple-Input-Multiple-Output (MIMO) cellular networks is introduced. It is based on the Poisson Point Process (PPP)-based abstraction for modeling the spatial locations of the Base Stations (BSs) and it exploits results from stochastic geometry to characterize the distribution of the other-cell interference. The framework is applicable to spatial multiplexing MIMO systems with an arbitrary number of antennas at the transmitter \( N_t \) and at the receiver \( N_r \). It is shown that the proposed framework leads to easy-to-compute integral expressions, which provide insights for network design and optimization. The accuracy of the mathematical analysis is substantiated through extensive Monte Carlo simulations for various MIMO cellular networks setups.

2.3.1 Introduction

As discussed in Section 2.1, the PPP-based abstraction model is now commonly used to the analysis and design of wireless networks due to its mathematical tractability. On the other hand, with the exception of a few papers, e.g., [2.20], the mathematical analysis is limited to single-antenna BSs and to single-antenna MTs.

By extending analysis in section 2.1 and [2.27]to multi-antenna networks, the technical contribution of this section is threefold:

1) we provide an exact closed-form expression of the Characteristic Function (CF) of the aggregate other-cell interference at the Mobile Terminal (MT);
2) by using the Gil-Pelaez inversion theorem[2.14], we provide an exact expression of the average pairwise frame error probability \( \text{AEP}_p^{(F)} \), which is averaged over both the fading distribution and all BSs deployments. From \( \text{AEP}_p^{(F)} \), the average frame error probability (AFEP) is obtained by using the Nearest Neighbor (NN) approximation[2.30];
3) asymptotic frameworks are proposed to provide insights on the achievable error performance as a function of important system parameters.

2.3.2 System Model

Similar to Section 2.1, in downlink cellular networks, a probe \( N_r \)-antenna MT is located at the origin of a bi-dimensional plane and the \( N_t \)-antenna BSs are modeled as points of a homogeneous PPP \( \Psi \) of density \( \lambda \). The distance from the \( i \)th BS to the MT is denoted by
\( r_i \) for \( i \in \Psi \). The MT is assumed to be tagged to the nearest BS. The serving BS is denoted by \( \text{BS}_0 \), and its distance from the MT is denoted by \( r_0 \), which is a RV with PDF

\[
f_{r_0}(\xi) = 2\pi\lambda \exp\left(-\pi\lambda\xi^2\right)
\]

[2.2]. The set of interfering BSs \( i \in \Psi \setminus \{\text{BS}_0\} \) is still a homogeneous PPP [2.29], which is denoted by \( \Psi^{(0)} \). \( \Psi^{(0)} \) has density \( p\lambda \), where \( 0 < p \leq 1 \) is the activity factor denoting the probability that an interfering BS \( i \in \Psi^{(0)} \) transmits in the same frequency band as \( \text{BS}_0 \). The setup with \( p = 1 \) corresponds to the full frequency reuse case [2.2], [2.3].

In the depicted downlink MIMO cellular network model, the signal received at the MT is as follows:

\[
y = \sqrt{\frac{E}{N}}r_0^{-\beta}H_0s_0 + \sqrt{\frac{E}{N}} \sum_{i\in\Psi^{(0)}} r_i^{-\beta}H_i s_i + n
\]

(2.3.1)

where \( x \in \mathbb{C}^{N_r \times 1} \) is the useful signal transmitted by \( \text{BS}_0 \), \( i_{agg}(r_0) \in \mathbb{C}^{N_r \times 1} \) is the aggregate other-cell interference and \( n \in \mathbb{C}^{N_r \times 1} \) is the Additive White Gaussian Noise (AWGN) with i.i.d. \( n^{(r)} \sim \mathbb{C}\mathcal{N}(0, N_0) \). More specifically: \( E \) is the BSs transmit-energy per transmission; \( s_0 \) is the vector of information symbols emitted by \( \text{BS}_0 \), where \( s_0^{(t)} \in \mathbb{M} \) and the set \( \mathbb{M} \) has \( M \) modulated symbols denoted by \( \mu_\chi \in \mathbb{C} \); \( H_0 \in \mathbb{C}^{N_r \times N_t} \) is the channel matrix of the \( \text{BS}_0 \)-to-MT link; \( b > 1 \) denotes the amplitude path-loss exponent. A similar notation is adopted for all the interfering channels of \( i_{agg}(\cdot) \).

At the MT, an interference-unaware Maximum-Likelihood (ML)-optimum demodulator is considered. It is assumed to have perfect Channel State Information (CSI) of the \( \text{BS}_0 \)-to-MT link, while ignoring the other-cell interference. The decision metric \( \Lambda(\cdot) \) can be formulated as [2.24]:

\[
\Lambda(\hat{s}_0) = \|y - \sqrt{\frac{E}{N}}r_0^{-\beta}H_0s_0\|^2 \propto r_0^{-2b} E \frac{U + 2r_0^{-b} E \text{Re}\{I(r_0)\} + 2r_0^{-b} E \text{Re}\{N\}}{N_t^2} 
\]

(2.3.2)

where \( \Lambda_0 = s_0 - \hat{s}_0 \) and:

\[
I(r_0) = \sum_{r=1}^{N_r} i_{agg}^{(r)}(r_0) \sum_{i=1}^{N_i} (H_0^{(r,i)}(t_0)^A)^2, \quad U = \sum_{r=1}^{N_r} \sum_{i=1}^{N_i} H_0^{(r,i)}(t_0)^A, \quad N = \sum_{r=1}^{N_r} \sum_{i=1}^{N_i} (H_0^{(r,i)}(t_0)^A)^2
\]
Assume Rayleigh channel model, $|H_0^{(r,t)}|^2 \sim \text{Gamma}(1, \Omega)$ and the channel phases are uniformly distributed, then $H_0^{(r,t)} \sim \mathcal{CN}(0, \Omega)$ for $t=1,2,\ldots,N_t$ and $r=1,2,\ldots,N_r$. Thus, by conditioning upon $\Lambda_0$, we have $\sum_{t=1}^{N_t} H_0^{(r,t)} \Lambda_0(t) \sim \mathcal{CN}_{|\Lambda_0|} \left(0, \Omega \|\Lambda_0\|^2\right)$. This implies that $U \sim \text{Gamma} \left(N_r, \Omega N_r \|\Lambda_0\|^2\right)$.

### 2.3.3 Characteristic Function of Interference

In this section, we provide a closed-form expression of the CF of $I(\cdot)$ in (2.3.2), which is the other-cell interference at the output of the demodulator.

**Theorem 2.3.1**

Let the downlink channels $H_i$ for $i \in \Psi^{(w)}$ be i.i.d. Rayleigh distributed, with $E \left(|H_i^{(r,t)}|^2\right) = \Omega$.

By conditioning upon $r_i$ and upon $U$ given in (2.3.2), a closed-form expression of the CF of $I(\cdot)$ in (2.3.2), which is the other-cell interference at the output of the demodulator, is provided as follows:

$$\Phi_I(\omega; r_i, U) = \Phi_I(\omega; r_0, U) = \exp \left\{-\lambda \pi r_i^2 T \left(|\omega|^2 r_0^{-2b} E U\right)\right\}$$

(2.3.3)

with $T(x) = p \bar{T}(x) - p$ and:

$$\bar{T}(x) = E_{s_i} \left\{ _1 F_1 \left(-\frac{1}{b}; 1 - \frac{1}{b}; -\frac{x}{4} N_r \Omega\right) \right\}$$

(2.3.4)

where $p F_q \left(a_1, \ldots, a_p; b_1, \ldots, b_q; \cdot\right)$ is the generalized hypergeometric function and the expectation $E_{s_i} \{\cdot\}$ computed over the $N_t$-tuple of information symbols $s_i \in \mathcal{C}^{N_t \times 1}$ can be formulated with $g(\cdot)$ being a generic function and $K > 0$ being a positive constant as follows:

$$E_{s_i} \left\{ g \left(\frac{K}{N_t} \|s_i\|^2\right) \right\} = \frac{1}{M^{N_t}} \sum_{x_1 \in 1} \sum_{x_2 \in 1} \cdots \sum_{x_N \in 1} g \left(\frac{K}{N_t} \sum_{t=1}^{N_t} |\mu_{t,x}|^2\right)$$

(2.3.5)

**Proof:** The proof is similar as [2.27]. The only difference is that $Z_i$ in $I_{agg}(\cdot)$ in [2.27] needs to be replaced by $\tilde{z}_i$ in $I(\cdot)$. In addition, and similar to $Z_i$, the RVs $\tilde{z}_i$ are still circularly symmetric and i.i.d. for $i \in \Psi^{(w)}$. As a consequence, the CF of $I(\cdot)$ can be formulated as shown in [2.27], by simply replacing $\text{Re} \left\{ Z_i \right\}$ with $\text{Re} \left\{ \tilde{z}_i \right\}$. Accordingly, the moments of...
$z_{i}^{(re)}$ conditioned upon $H_0$ and $\Delta_0$ need to be computed. Let us denote these conditioned moments by $\eta_q (H_0, \Delta_0) = E_{z_{i}\mid H_0, \Delta_0} \left\{ (z_{i})^{2q} \right\}$. Since $H_i^{(re)} \sim \text{CN} (0, \Omega)$, $\text{Re} \left\{ H_i^{(re)} \right\} \sim \text{CN} (0, \Omega / 2)$. Thus, $z_{i}^{(re)}$ turns out to be a Gaussian RV by conditioning upon $H_0$, $\Delta_0$ and $s_i$, which we denote by $z_{i}^{(re)} = N_{H_0, \Delta_0, s_i} \left( 0, (\Omega / 2) U (1 / \Delta_0) \sum_{i=1}^{N_1} |s_i|^2 \right)$. Then the proof can be proceeded as [2.27] by following from the moments of Gaussian RVs [2.32]

$$\eta_q (H_0, \Delta_0) = \eta_q (U) = E_{s_i} \left\{ (-1)^q \frac{\sqrt{\pi}}{\Gamma (1/2 - q)} \left( \frac{1}{\Delta_0} \sum_{i=1}^{N_1} |s_i|^2 \right)^q \Omega^q \right\}$$

and from the series representation of the generalized hypergeometric function [2.31], and finally from the identities $\sum_{i=1}^{N_1} |s_i|^2 = \|s_i\|^2$ and $F(z, \alpha; \beta; x) = F_1 (\alpha; \beta; -x)$. □

### 2.3.4 Error Performance

**APEP**

By capitalizing on the CF, APEP is computed, which is defined as the probability that transmitted vector $s_0$ is decoded as $\hat{s}_0 = s_0 \neq s_0$, by assuming that $s_0$ and $\bar{s}_0$ are the only two information vectors possibly being transmitted.

**Theorem 2.3.2**

Let $SNR = E / N_0$. Then, APEP can be formulated as

$$APEP (\Delta_0) = \text{Pr} \{ \Delta (s_0) < 0 \} = APEP N (\Delta_0) + APEP NI (\Delta_0)$$

which are defined as:

$$APEP N (\Delta_0) = b \sqrt{SNR / N_t} \int_0^{\infty} e^{\frac{-x^2}{2}} Q_N (x; \Delta_0, SNR / N_t, b) dx$$

$$APEP NI (\Delta_0) = (2\pi)^{-1} \times \int_0^{\infty} (1 - x) e^{-x} e^{-4\pi^2 x SNR} \left[ 1 - e^{-\gamma (N_t, x)} \right] Q_N (x; \Delta_0) dx dy$$

where:

$$Q_N (x; \Delta_0, SNR / N_t, b) = E_U \left\{ \sqrt{U} \exp \left( -\frac{SNRU}{4N_t x^2 b} \right) \right\}$$

$$Q NI (x; \Delta_0) = E_U \left\{ \sin \frac{1}{2} \sqrt{U} x^2 \right\}$$
**Proof:** The proof is similar to [2.27]. By applying the Gil-Pelaez inversion theorem [2.14] to (2.3.2), APEP\(^{(F)}\) conditioned upon \(H_0\) and \(r_0\) can be formulated as follows:

\[
\text{APEP}^{(F)}(\Lambda_0; H_0, r_0) = \text{APEP}^{(F)}(U, r_0)
\]

\[
= \frac{1}{2} \int_0^{\infty} \sin \left( \frac{r_0 - b}{2} \sqrt{\frac{E}{N_r}} |\omega| \right) \Phi_{N_i}(|\omega|; U) \Phi_{r_i}(|\omega|; r_0, U) \frac{d|\omega|}{|\omega|}
\]

where: i) \(N^{(m)} = \text{Re}\{N\} \sim N(0, (N_0/2)U)\) follows from the AWGN assumption and from (2.3.2); ii) \(I^{(m)} = \text{Re}\{I\}\) is defined in (2.3.2); iii) \(\Phi_{N_i}(|\omega|; U) = \Phi_N(|\omega|; U)\) and \(\Phi_{r_i}(|\omega|; r_0, U) = \Phi_I(|\omega|; r_0, U)\), since both \(N\) and \(I\) are circularly symmetric RVs; iv) \(\Phi_N(|\omega|; U) = \exp\left(-\left(1/4\right)|\omega|^2 N_0 U\right)\), since it is a Gaussian RV [2.33]; and v) \(\Phi_I(\cdot; \cdot)\) is given in (2.3.3). The rest of the proof is the same as in [2.27]. □

With the aid of [2.34] and of the Kummer's transformation

\[
_1F_1(1-m; 3/2, x)\exp\{-x\} = _1F_1(m+1/2; 3/2, -x)
\]

for arbitrary \(K_1 > 0\) and \(K_2 > 0\) positive constants and a Gamma RV \(U \sim \Gamma(N_r, \Omega_N, \|\Lambda_0\|^2)\), we have close-from expressions for (2.3.7) as follows:

\[
E_U\left\{\sqrt{U}\exp\{-K_1U\}\right\} = \frac{N_r^N \Gamma\left(N_r + \frac{1}{2}\right)}{\left(\Omega_N, \|\Lambda_0\|^2\right)^N \Gamma\left(N_r\right)} \left(K_1 + \left(\Omega_N, \|\Lambda_0\|^2\right)^{N_r, +1/2}\right)^{N_r, +1/2}, \quad (2.3.8)
\]

\[
E_U\left\{\sin\left(K_2\sqrt{U}\right)\right\} = \frac{\sqrt{\Omega_N, \|\Lambda_0\|^2} \Gamma\left(N_r + \frac{1}{2}\right)}{\Gamma\left(N_r\right)} K_2 F_1\left(N_r, +1/2; 3/2, -1/4 \Omega_N, \|\Lambda_0\|^2 K_2^2\right)
\]

We then provide simplified mathematical frameworks in two limiting operating conditions: 1) noise-limited, i.e., \(\text{SNR} \rightarrow 0\), and 2) interference-limited, i.e., \(\text{SNR} \rightarrow +\infty\), cellular networks, where \(\text{SNR} = E / N_0\) denotes the Signal-to-Noise-Ratio (SNR). As discussed in[2.3], typical cellular networks operate in the interference-limited regime.

*Corollary 2.3.1*

For noise-limited (\(\text{SNR} \rightarrow 0\)) and interference-limited (\(\text{SNR} \rightarrow +\infty\)) networks, asymptotic frameworks can be drawn from (2.3.6):
These asymptotic frameworks can better provide insights on the error performance as a function of important system parameters.

For example, it can be noted that error probability is independent of both SNR and BS density $\lambda$ in interference-limited networks. On the other hand, in noise-limited networks, the error probability decreases by increasing SNR and BS density $\lambda$ respectively. Also in interference-limited regime $\text{APEP}^{(F)}$ decreases by increasing the path-loss exponent $b$ and $\text{APEP}^{(F)}$ decreases by decreasing the activity factor $p$. All the trends and insights inferred from mathematical frameworks can be substantiated by Monte Carlo simulations, which we will show in the next section.

**AFEP**

Finally, based on the $\text{APEP}^{(F)}$ in (2.3.6) and on the NN approximation[2.30], AFEP can be approximated as follows:

$$
\text{AFEP} \approx \left(1/M^{N_t}\right) \sum_{\zeta=1}^{M^{N_t}} N_{\text{min}(\zeta)}^{(x)} \text{APEP}^{(F)} \left(\|\Delta_{\text{min}(\zeta)}\|^2\right)
$$

(2.3.9)

where: i) $\|\Delta_{\text{min}(\zeta)}\|^2 = \min_{\mu_{\zeta}} \left\{\|\mu_{\zeta} - \mu_{\zeta^*}\|^2 : \mu_{\zeta^*} \in M^{N_t} \right\}$ is the minimum Euclidean distance among all pairs $(\mu_{\zeta}, \mu_{\zeta'})$ of the $N_t$-dimensional constellation diagram and ii) $N_{\text{min}(\zeta)}^{(x)}$ is the number of nearest neighbors of $\mu_{\zeta^*}$. For example, considering the standard square QAM with $M$ symbols, we have $(\|\Delta_{\text{min}}\|^2, N_{\text{min}}^{(x)}) = (4, 2)$ if $(M = 2, N_t = 2)$, $(\|\Delta_{\text{min}}\|^2, N_{\text{min}}^{(x)}) = (2, 4)$ if $(M = 4, N_t = 2)$, $(\|\Delta_{\text{min}}\|^2, N_{\text{min}}^{(x)}) = (0.4, 6)$ if $(M = 16, N_t = 2)$, $(\|\Delta_{\text{min}}\|^2, N_{\text{min}}^{(x)}) = (2, 8)$ if $(M = 4, N_t = 4)$. It is worth noting that $N_{\text{min}}^{(x)} \in \{4, 5, 6, 7, 8\}$ if $(M = 16, N_t = 2)$. 
2.3.5 Numerical Results

We validate in this section the mathematical frameworks against Monte Carlo simulations, which are obtained by using the procedure described in [2.3], and study the impact of different downlink MIMO cellular setups.

Before that, we first validate the PPP-based abstraction model by comparing it with modeling the BS locations via grid-based model. A similar comparison is available in [2.2] for the coverage probability. We provide in this section the comparison for AFEP. Fig.1 shows that the PPP-based abstraction model ("solid lines") provides a worst estimate of the error probability compared to the grid-based abstraction model ("dash curves"), since interfering BSs may be arbitrarily close to each other.

On the other hand, the PPP-based abstraction model provides tractability and performance trends as a function of system parameters can be inferred from the resulting mathematical frameworks. We observe a good accuracy of the obtained frameworks (depicted in curves with colored markers) compared with the Monte Carlo simulations (depicted in “black” curves).

Figs. 2.3.1 – 2.3.6 all support the claim that AFEP is independent of SNR in the interference-limited regime. Fig. 2.3.1 confirms that AFEP decreases by decreasing the activity factor \( p \). Fig. 2.3.2 shows that by increasing the path-loss exponent \( b \), AFEP decreases in the interference-limited regime while increases in the noise-limited regime. Fig. 2.3.3 confirms that having multiple-antenna at the receiver leads to an improvement of the ASEP/AFEP. However, no receive diversity gain is obtained in the interference-limited regime. The rate of the MIMO system is defined as \( \text{Rate} = N_i \log_2(M) \) bits per channel use (bpcu). Conditioned on a consistent rate per channel use, Fig. 2.3.5 shows that increasing \( N_i \) provides a better AFEP but the performance difference as a function of \( N_i \) is smaller compared to \( N_r \). With symmetric antenna setup \( N_i = N_r \), Fig. 2.3.6(a) shows the AFEP by assuming that the density of BSs \( \lambda \) is kept the same but the number of BSs antennas \( N_i \) is different and Fig. 2.3.6(b) shows the AFEP by assuming that the density of BSs antennas \( \lambda_0 = N_i \lambda \) is kept the same and the BSs density \( \lambda \) depends on \( N_i \). The trends shown in Fig. 2.3.6 allows one to reduce the density of BSs by increasing \( N_i \) without performance degradation in the
interference-limited regime and with a small performance degradation in the noise-limited regime.

![Figure 2.3.1](image1.png)

Figure 2.3.1 AFEP of a $N_t \times N_r = 2 \times 2$ system against the transmit SNR. Setup: 16QAM, $\lambda = 10^{-5}$ and $b = 3$.

![Figure 2.3.2](image2.png)

Figure 2.3.2 AFEP of a $N_t \times N_r = 2 \times 2$ system against the transmit SNR. Setup: 16QAM, $\lambda = 10^{-5}$ and $p = 10^{-3}$. 
Figure 2.3.3 AFEP of a $N_r \times N_r = 2 \times 2$ system against the transmit SNR. Setup: 16QAM, 

$$b = 3 \text{ and } p = 10^{-3}.$$ 

Figure 2.3.4 AFEP of a $N_r \times N_r = 2 \times N_r$ system against the transmit SNR. Setup: 4QAM, 

$$\lambda = 10^{-3}, \; b = 3 \text{ and } p = 10^{-3}.$$
Figure 2.3.5 AFEP of a $N_t \times N_r = N_t \times 2$ system with $\text{Rate} = 4$ bpcu against the transmit SNR. Setup: $b = 3$, $\lambda = 10^{-5}$, $b = 3$ and $p = 10^{-3}$.

Figure 2.3.6 AFEP of a system with $\text{Rate} = 4$ bpcu against the transmit SNR. Setup: (a) $b = 3$, $\lambda = 10^{-5}$, $p = 10^{-3}$. (b) $b = 3$, $\lambda_0 = 10^{-5}$, $p = 10^{-3}$. 
2.3.6 Conclusion
In this section, a new mathematical framework to the computation of the average error probability of downlink MIMO cellular networks have been proposed and have been substantiated with the aid of Monte Carlo simulations. Their analysis has revealed important performance trade-offs that may emerge depending on the SNR operating regime, the channel attenuation parameters and the number of antennas available at the BSs and MT.

2.4 Stochastic geometry modeling and analysis of the error probability of single-tier SISO cellular networks: the Equivalent-in-Distribution (EiD) based approach

2.4.1 Introduction
As discussed in the previous section, the PPP-based approach provides a tractable mathematical analysis and is as accurate as other abstraction models [2.2]. In general, two main performance metrics have been studied to date, i.e., the outage probability and the average rate [2.1], [2.2], [2.3]. Less attention has been given, on the other hand, to the computation of the ASEP, which is, however, a relevant figure of merit to wireless systems analysis and design. In fact, it is directly related to the bit, packet, block and frame error probabilities, which are important performance metrics to the design of cellular networks [2.52].

Indeed, the framework introduced in Section 2.2 and 2.3, based on the Gil-Peleaz inversion approach as well as the APEP, providing the ASEP using the nearest neighbor approximation, is, however, not exact. In the following three sections, we introduce a new mathematical methodology for the computation of the error probability of downlink MIMO cellular networks. The proposed approachcapitalizes on the so-called Equivalent-in-Distribution (EiD)-based method, by finding EiD representations of the aggregate other-cell interference, which is formulated as a linear combination of conditionally Gaussian Random Variables (RVs). With the aid of this mathematical formulation, the error probability is computed by first conditioning upon the non-Gaussian RVs and by then removing the conditioning. The usefulness of this approach lies in the possibility of obtaining exact mathematical expressions in the presence of non-Gaussian distributed interference. In Section 2.4, we first introduce the EiD-based framework by only considering a single-tier SISO cellular network, while the framework is extended to studying a variety of MIMO setups in Section 2.5. In Section 2.6, we further extend the application of the EiD-based approach
from the conventional single-tier cellular network to multi-tier heterogeneous cellular networks.

Compared to Section 2.2 and 2.3, the new approach: i) is not based on the Gil-Pelaez inversion theorem; ii) is applicable to many MIMO schemes (not just to spatial multiplexing); and iii) provides, in many cases, exact integral expressions of the error probability. In the presence of other-cell interference and noise, the error probability is formulated in terms of a two-fold integral. The framework is shown to reduce to the computation of a single integral in interference-limited cellular networks.

The following notations are used in this section: \( \mathbb{N}^+ \) denotes the set of positive integers. \( \mathbb{E} \{ \} \) is the expectation operator. \( j = \sqrt{-1} \) is the imaginary unit. \( \text{CN}(\mu, \sigma^2) \) is a complex Gaussian Random Variable (RV) with mean \( \mu \) and variance \( \sigma^2 \). \( \mathbb{R} \{ \} \) and \( \mathbb{I} \{ \} \) are real and imaginary part operators. \( \Phi_z(\omega) = \mathbb{E} \{ \exp \left( j \omega \mathbb{R} \{ z \} + \omega \mathbb{I} \{ z \} \right) \} \) is the Characteristic Function (CF) of the complex RV \( z = \mathbb{R} \{ z \} + j \mathbb{I} \{ z \} \), where \( \omega = (\omega_1, \omega_2) \). \( || \) is the absolute value of vectors and complex RVs. \( x! \) is the factorial of \( x \). \( \Gamma(x) = \int_0^\infty t^{x-1} \exp \{-t\} dt \) is the Gamma function. \( (x)_k = \Gamma(x+k)/\Gamma(x) \) is the Pochhammer symbol. \( x \models y \) denotes that \( x \) and \( y \) are EiD. \( \mathbb{M}_\lambda(x) = \mathbb{E} \{ \exp \{-sx\} \} \) is the Moment Generating Function (MGF) of \( x \). \( F_{pq} \left( \cdot; \cdot; \cdot \right) \) is the generalized hypergeometric function [Ch. 5, Eq. (2)]. \( \text{erf} \left( x \right) = \frac{2}{\sqrt{\pi}} \int_0^x \exp \left\{-t^2\right\} dt \) is the error function.

### 2.4.2 System model

A bi-dimensional downlink cellular network deployment as depicted in [2.2] is studied, where a probe single-antenna Mobile Terminal (MT) is located at the origin and the single-antenna Base Stations (BSs) are modeled as points of a homogeneous PPP (\( \Psi \)) of density \( \lambda \). The MT is served by the nearest BS (\( \text{BS}_0 \)). Their distance is denoted by \( r_0 \), which is a RV having Probability Density Function (PDF) equal to \( f_{r_0}(\xi) = 2\pi \lambda \xi \exp \{-\pi \lambda \xi^2\}[2.2] \). Similar to [2.2], a per-cell random fractional frequency reuse is considered, where \( 0 \leq p \leq 1 \) denotes the probability that the generic interfering BS transmits in the same frequency band as \( \text{BS}_0 \).
According to [2.47], the set of interfering BSs (Ψ_p(0)) is a homogeneous PPP of density \( p \lambda \).

The distance from the \( i \)th interfering BS to the MT is denoted by \( r_i > r_0 \) for \( i \in \Psi_p(0) \).

In the depicted downlink cellular network model, the (complex) signal received at the MT can be formulated as follows:

\[
y = \sqrt{E_0} s_0 h_0 + \sum_{i \in \Psi_p(0)} \sqrt{E_i} s_i h_i + n = x + i_{agg}(r_0) + n \tag{2.4.1}
\]

where \( x = \sqrt{E_0} s_0 h_0 \) is the useful signal from BS_0, \( i_{agg}(r_0) = \sum_{i \in \Psi_p(0)} \sqrt{E_i} s_i h_i \) is the other-cell interference, which depends on \( r_i \) since \( r_i > r_0 \), \( n \sim \text{CN}(0,N_0) \) is the Additive White Gaussian Noise (AWGN), \( E_0 \) is the average symbol energy of BS_0, \( s_0 = a_0 \exp\{j\theta_0\} \) is the symbol transmitted by BS_0 and \( h_0 = (1/r_0^b) \alpha_0 \exp\{j\phi_0\} \) is the channel impulse response of the BS_0-to-MT link, with \( b_0 > 1 \) denoting the path-loss exponent, \( \alpha_0 \) denoting the fading envelope, and \( \phi_0 \) being a uniformly distributed RV in \([0,2\pi]\). For ease of illustration, Rayleigh fading is assumed. Hence, \( \alpha_0^2 \) is an exponential RV with parameter \( \Omega_0 = \text{E}\{\alpha_0^2\} = 1[2.24] \). A similar notation is used for \( i_{agg}(\cdot) \). In particular, we assume that all the BSs transmit the same average energy per symbol (\( E_0 = E_i = E \)) and that all the channels are independent and identically distributed having the same path-loss exponents (\( b_0 = b_i = b \)) and the same fading parameters (\( \Omega_0 = \Omega_i = \Omega \)). A general bi-dimensional modulation scheme is considered, whose \( M \) equiprobable symbols are \( s^{(m)} = a^{(m)} \exp\{j\theta^{(m)}\} \) for \( m = 1,2,\ldots,M \). Then, \( s_0 \in \{s^{(m)}\} \) and \( s_i \in \{s^{(m)}\} \) for \( i \in \Psi_p(0) \). The constellation diagram is assumed to have unit average energy, i.e., \( (1/M) \sum_{m=1}^{M} |s^{(m)}|^2 = 1 \), thus \( \text{E}\{|s_0|^2\} = \text{E}\{|s_i|^2\} = 1 \).

At the MT, the optimal demodulator in AWGN is used [2.24]:

\[
\hat{s}_0 = \arg\min_{s_0 \in \{s^{(m)}\}, m=1,2,\ldots,M} \left\{ \left| y - \sqrt{E_0} h_0 s_0 \right|^2 \right\} \tag{2.4.2}
\]

where \( \hat{s}_0 \) is the estimate of the actual transmitted symbol \( s_0 \).

In this section, we are interested in computing the ASEP of the demodulator in (2.4.2), which is the probability that \( \hat{s}_0 \neq s_0 \).
2.4.3 Characteristic function of the interference

In this section, the CF of \( i_{agg}(\cdot) \) is presented again to increase the readability. For simplicity, we use the notation \( z_i = a_i \alpha_i \exp\{ j\theta_i \} \exp\{ j\phi_i \} \). Hence, \( i_{agg}(\cdot) \) simplifies to

\[
i_{agg}(r_0) = \sum_{i \in \mathcal{N}(r_0)} \sqrt{E}(z_i / r_0^b) . \]

Then the CF of \( i_{agg}(r_0) \) given \( r_0 \) is \( \Phi_{agg}(\omega; r_0) = \Phi_{agg}(\omega^2; r_0) \), as follows:

\[
\Phi_{agg}(\omega^2; r_0) = \exp\left\{-\frac{p^2 \pi r_0^2}{M} \sum_{m=1}^{M} \sum_{q=1}^{\infty} \Gamma_q \left| z_q \left( \frac{\omega^2}{r_0^2} \right) \right|^q \right\} \quad (2.4.3)
\]

where \( \Gamma_q = (-4)^{-q} (q!)^{-1} (-1/b)_q \left( (1 - 1/b)_q \right)^{-1} \).

2.4.4 The EiD-Based Approach

From (2.4.3), it is apparent that the aggregate other-cell interference \( i_{agg}(\cdot) \) does not follow a common probability distribution. In particular, \( i_{agg}(\cdot) \) does not follow either a Gaussian distribution, as in AWGN channels [2.24], or a symmetric alpha stable distribution, as in ad hoc networks [2.10], for which tractable frameworks to the computation of the ASEP exist. This makes the mathematical computation of the ASEP of cellular networks difficult. If a binary modulation scheme is considered, i.e., \( M = 2 \), the ASEP can be obtained by exploiting the Gil-Pelaez inversion theorem. It provides, however, either approximations or bounds for \( M > 2 \). In this subsection, we introduce a new mathematical approach that leads to exact expressions of the ASEP of cellular networks for arbitrary bi-dimensional modulations with \( M \geq 2 \).

The proposed approach finds inspiration from the methodology recently introduced in [2.10], which is applicable to decentralized wireless networks. To better introduce our approach, we briefly summarize the methodology adopted in [2.10]. The system model in (2.4.1) reduces to that of [2.10] by letting \( r_0 = 0 \), i.e., by replacing \( i_{agg}(r_0) \) with:

\[
i_{agg}(\cdot) = i_{agg}(r_0 = 0) = B_{agg}^{1/2} G_{agg} \quad (2.4.4)
\]

where \( i_{agg}(\cdot) \) follows a symmetric alpha stable distribution, \( B_{agg} \) is a real stable RV totally skewed to the right and \( G_{agg} \) is a complex Gaussian RV with zero mean and variance available in [2.10].
By capitalizing on the EiD-based representation in (2.4.4), the authors of [2.10] propose a marginalization-based approach to the computation of the ASEP: 1) first, the ASEP conditioned upon \( h_0 \) and \( \text{B}_{agg} \) is computed by using formulas applicable to AWGN channels [2.24] and 2) then, the conditioning upon \( h_0 \) and \( \text{B}_{agg} \) is removed by computing the related integrals.

Inspired by this approach, in what follows we seek to answer the following question: Is it possible to develop an EiD-based representation of \( i_{agg}(r_0) \) for arbitrary values of \( r_0 \geq 0 \) in order to leverage a marginalization-based approach to the performance analysis of cellular networks? A positive answer to this question is provided in the following theorem:

**Theorem 2.4.1:**

Let \( i_{agg}(r_0) \) having CF \( \Phi_{i_{agg}}(\cdot; r_0) \) in (2.4.3). Let \( \text{B}_{agg}^{(q)} \) for \( q \in \mathbb{R}^+ \) be independent real RVs whose MGF is \( \mathcal{M}_{\text{B}_{agg}^{(q)}}(s) = \exp\{-s^q\} \). Let \( \text{G}_{agg}^{(q)} \) for \( q \in \mathbb{R}^+ \) be independent complex Gaussian RVs \( \text{G}_{agg}^{(q)} \sim \text{CN}(0, \sigma_q^2(r_0)) \) with:

\[
\sigma_q^2(r_0) = 4 \left[ p \lambda \pi r_0^2 Y_q \left( \frac{E \Omega}{r_0^2} \right)^q \frac{1}{M} \sum_{m=1}^{M} |s_m|^{2q} \right]^{1/q}.
\] (2.4.5)

The RVs \( \text{B}_{agg}^{(q)} \) and \( \text{G}_{agg}^{(q)} \) are independent for \( q \in \mathbb{R}^+ \). Then:

\[
i_{agg}(r_0) = \sum_{q=1}^{\infty} \sqrt{\text{B}_{agg}^{(q)} \text{G}_{agg}^{(q)}}
\] (2.4.6)

### 2.4.5 Performance Analysis

From Theorem 2.4.1, an EiD-based formulation of (2.4.1) is:

\[
y = \sqrt{E s_0} \alpha_0 \frac{\exp\{j\phi_0\}}{r_0^b} + \sum_{q=1}^{\infty} \sqrt{\text{B}_{agg}^{(q)} \text{G}_{agg}^{(q)}} + n
\] (2.4.7)

Conditioning upon \( s_0 \), \( h_0 \) and \( \text{B}_{agg}^{(q)} \) for \( q \in \mathbb{R}^+ \), \( y \) in (2.4.7) is conditionally-Gaussian. So, the ASEP of (2.4.2) can be computed from the classical definition of SINR in AWGN [2.24]:

Security: Public
\[
\text{SINR} \left( \alpha_0, r_0, \{B_{agg}^{(q)} \} \right) = \frac{E_x \left\{ \left[ E_{n_c[G_{agg}^{(q)}]} \left\{ y \right\} \right]^2 \right\} }{E_x \left\{ \left[ E_{n_c[G_{agg}^{(q)}]} \left\{ y \right\} \right]^2 \right\} } = \frac{E \alpha_0^2}{N_0 \frac{2^b}{r_0^2}} \left( 1 + \sum_{q=1}^{\infty} B_{agg}^{(q)} \sigma_q^2 (r_0) \right)^{-1} (2.4.8)
\]

where \( E_x \{ \} \) denotes the expectation computed only over \( X \).

By definition from (2.4.2), \( \text{ASEP} = \Pr \{ \hat{s}_0 \neq s_0 \} \). Based on the EiD-based representation in (2.4.7), the ASEP of bi-dimensional modulations can be formulated as the linear combination of integrals like the following [2.10]:

\[
J_{\text{SINR}} (\mu, \eta, \chi) = \frac{1}{\pi} \int_0^\mu M_{\text{SINR}} \left( \chi \frac{\sin^2 (\eta)}{\sin^2 (\theta)} \right) d\theta (2.4.9)
\]

where \( M_{\text{SINR}} (\cdot) = E_{\hat{s}_0, q, \{X_{agg}^{(q)} \}} \{ \exp \{ -s \text{SINR} \} \} \) is the MGF of the SINR in (2.4.8) and \((\mu, \eta, \chi)\) is a triplet of modulation-dependent parameters.

For example, the ASEP of square Quadrature Amplitude Modulation (QAM) is:

\[
\text{ASEP}_{\text{QAM}} = (4 / \delta) J_{\text{SINR}} (\mu_1, \eta_1, \chi_1) - (4 / \delta^2) J_{\text{SINR}} (\mu_2, \eta_2, \chi_2) (2.4.10)
\]

where \( \mu_1 = \pi / 2 \), \( \mu_2 = \pi / 4 \), \( \eta_1 = \eta_2 = \pi / 2 \), \( \chi_1 = \chi_2 = (3 / 2)(M - 1)^{-1} \) and \( \delta = (\sqrt{M} - 1) / \sqrt{M} \).

Let the SINR in (2.4.8). Then, \( J_{\text{SINR}} (\cdot, \cdot, \cdot) \) is:

\[
J_{\text{SINR}} (\mu, \eta, \chi) = (\mu / \pi) - \lambda \chi^2 \sin^2 (\eta) \int_0^{\infty} \Theta (x, y) dxdy (2.4.11)
\]

where

\[
\Theta (x, y) = \exp \left\{ - \left( \frac{E \Omega}{N_0} \right)^{-1} x^2 y \right\} \exp \left\{ - \lambda x Q (y) \right\} Y \left( \chi^2 \sin^2 (\eta) y \right) (2.4.12)
\]

\[
Q (y) = (p / M) \sum_{m=1}^{M} F_1 \left( \frac{1 - 1/b; 1 - 1/b; \left| s^{(m)} \right|^2 y}{} \right) - p + 1 (2.4.13)
\]

and

\[
Y (y) = \begin{cases} Y^{(\cdot)} (y) & \text{if } 0 \leq \mu \leq \pi / 2 \\ Y^{(\cdot)} (y) & \text{if } \mu > \pi / 2 \end{cases}
\]

with

\[
Y^{(\cdot)} (y) = \left( \exp \{ -y \} / 2 \right) \sqrt{\pi / y} \left[ 1 + \text{erf} \left( \sqrt{y \cot^2 (\mu)} \right) \right] .
\]
Typical cellular networks are interference-limited [2.3]. Let $J_{\text{SINR}}(\cdot, \cdot, \cdot)$ in (2.4.11). If $N_0 = 0$, then:

$$J_{\text{SINR}}(\mu, \eta, \chi) = \frac{\mu}{\pi} - \frac{\chi^2 \sin^2(\eta)}{\pi} \int_0^{\infty} Y\left(\chi^2 \sin^2(\eta) y\right) Q(y) \, dy$$

(2.4.14)

### 2.4.6 Numerical Results

In Fig. 2.4.1, some numerical examples are shown in order to substantiate the accuracy of the EiD-based approach. Monte Carlo simulations are obtained as described in [2.3]. At the MT, the demodulator in (2.4.2) is used. A good agreement between mathematical framework and simulations is observed. As suggested by (2.4.14), the ASEP is independent of $\lambda$ in the interference-limited regime (high $E / N_0$). Decreasing the frequency reuse factor $p$ leads to a better ASEP. However, the average rate decreases by decreasing $p$ [2.3]. Hence, a trade-off emerges. The impact of $b$ depends on $E / N_0$. In the interference-limited regime, the higher $b$ the better the ASEP.

Fig. 2.4.1: Validation of the EiD-based Approach
2.5 Extension of the Equivalent-in-Distribution (EiD) based approach to single-tier MIMO cellular networks

2.5.1 Introduction

In the present section, we introduce a new mathematical methodology for the computation of the error probability of downlink MIMO cellular networks based on the EiD representation of the other-cell interference similar as that proposed in Section 2.4. The mathematical framework introduced in Section 2.4, however, is applicable only to SISO cellular networks for transmission over Rayleigh fading channels. Its generalization to MIMO cellular networks and to other fading distributions is, however, not straightforward. In Section 2.4, in fact, the error probability is obtained by first computing the Cumulative Distribution Function (CDF) of the Signal-to-Interference-plus-Noise-Ratio (SINR) and by then applying the so-called CDF-based approach. This methodology is effective in Rayleigh fading, as the CDF of the power gain of the intended link is an exponential function, which is conveniently formulated for further analysis. It is known, on the other hand, that the generalization of CDF-based methods to other fading distributions is problematic [2.2], [2.3], [2.28]. Unfortunately, the equivalent power gain of the intended link of MIMO transmission schemes is not exponentially distributed even in Rayleigh fading channels [2.20]. As a consequence, a new mathematically tractable approach is necessary in this context.

In the present section, the limitations are overcome by introducing a new mathematical framework that is based on the computation of the Moment Generating Function (MGF) of the equivalent power gain of the intended link, which makes the EiD-based approach applicable to a number of MIMO arrangements for transmission over Rayleigh fading channels. As a byproduct, we show that the proposed approach is applicable to SISO cellular networks for transmission over Nakagami-m fading channels. Compared to [2.20], the approach in this section is different since it does not exploit stochastic ordering. Our approach: i) is not based on the Gil-Pelaez inversion theorem; ii) is applicable to many MIMO schemes (not just to spatial multiplexing); and iii) provides, in many cases, exact integral expressions of the error probability. In the presence of other-cell interference and noise, the error probability is formulated in terms of a two-fold integral. The framework is shown to reduce to the computation of a single integral in interference-limited cellular networks. Also, a simple closed-form expression is introduced, which provides meaningful insights on the impact of various system parameters that determine the achievable performance of MIMO cellular networks.
In this section, the following notation is used. $z^*$, $\vert z \vert$ and $\arg \{ z \}$ denote conjugate, modulus and phase operators of a complex number $z$. $z_1 \propto z_2$ denotes that $z_1$ is directly proportional to $z_2$. $\mathbb{C}$ denotes the field of complex numbers. $x \in S^{K \times 1}$ denotes a $K \times 1$ column-vector with entries belonging to the set $S$. The $k$th entry is denoted by $x^{(k)}$. $X \in S^{K \times L}$ denotes a $K \times L$ matrix with entries belonging to the set $S$. The $(k,l)$th entry is denoted by $X^{(k,l)}$. $X^H$ denotes the Hermitian of $X$. $\{ \} \text{card} S$ denotes the cardinality of the set $S$. $\| x \|$ denotes the norm of vector $x$. $\| X \|$ denotes the Frobenius norm of matrix $X$. $j = \sqrt{-1}$ denotes the imaginary unit. $\mathbb{CN}(\mu, \sigma^2)$ and $\mathbb{N}(\mu, \sigma^2)$ denote a complex and a real Gaussian distribution with mean $\mu$ and variance $\sigma^2$. The notation $\mathbb{CN}_X(\mu, \sigma^2)$ and $\mathbb{N}_X(\mu, \sigma^2)$ is used for Gaussian RVs conditioned upon the RV $X$. $U(a,b)$ denotes a uniform distribution in $(a,b)$. $\mathbb{G}(m, \Omega)$ denotes a Gamma distribution with fading parameter $m$ and mean square value $\Omega$. $\chi_d^2$ denotes a Chi-Square distribution with $d$ degrees of freedom. $Y = X_1 / X_2 \sim F(d_1,d_2)$ denotes a F-distribution with parameters $d_1$ and $d_2$, where $X_1 \sim \chi_{d_1}^2 / d_1$ and $X_2 \sim \chi_{d_2}^2 / d_2$. $(\cdot)_q$ denotes the Pochhammer symbol, where $q$ is a non-negative integer. $(\cdot)!$ and $(\cdot)!!$ denote factorial and double factorial operators. $\text{Re}\{\cdot\}$ and $\text{Im}\{\cdot\}$ denote real and imaginary part operators. $E\{\cdot\}$ denotes the expectation operator. $f_X(\cdot)$ denotes the Probability Density Function (PDF) of RV $X$. $\Phi_X(\omega) = E\{\exp\{j(\omega_1 X^{(re)} + \omega_2 X^{(im)})\}\}$ denotes the Characteristic Function (CF) of a complex RV $X = \text{Re}\{X\} + j\text{Im}\{X\} = X^{(re)} + jX^{(im)}$, where $\omega = (\omega_1, \omega_2)$. For simplicity, the short-hand $\Phi_X(\omega) = E\{\exp\{j\omega X\}\}$ is used. $M_X(s) = E\{\exp(-sX)\}$ denotes the MGF of a real RV $X$. $\text{Pr}\{\cdot\}$ denotes probability. $\binom{\cdot}{\cdot}$ denotes the binomial coefficient. $X \overset{d}{=} Y$ denotes that the RVs $X$ and $Y$ are EiD. $\Gamma(\cdot)$ is the Gamma function. $\Gamma(\cdot, \cdot)$ is the upper-incomplete Gamma function. $\rho F_q(a_1, \ldots, a_p; b_1, \ldots, b_q; \cdot)$ is the generalized
hypergeometric function \[2.21\]. \(J_v(\cdot)\) is the Bessel function of the first kind \[2.35\].

\[
G_{p,q}^{a,b}\left( \cdot \middle| \begin{pmatrix} a_p \\ \vdots \\ b_q \end{pmatrix} \right)
\]

is the Meijer G-function \[2.22\].

### 2.5.2 System model

We consider a bi-dimensional downlink cellular network deployment, where a typical multi-antenna Mobile Terminal (MT) is located at the origin and the multi-antenna BSs are modeled as points of a homogeneous PPP \(\Psi\) of density \(\lambda\). The number of antennas at each BS and at the MT is denoted by \(N_f\) and \(N_r\), respectively. The other setups are similar as those depicted in Section 2.4.

In this MIMO cellular network model, data transmission occurs in frames of \(N_s\) time-slots each. The signal received at the MT in the \(\tau\) th time-slot can be formulated as follows (\(\tau = 1, 2, \ldots, N_s\)):

\[
y(\tau) = \sqrt{E/N_f} r_0 H_{0} s_0(\tau) + \sqrt{E/N_r} \sum_{\iota \in \Psi(\cdot)} r_{\iota}^* H_{\iota} s_{\iota}(\tau) + n(\tau)
\]  

(2.5.1)

where \(y(\cdot) \in \mathbb{C}^{N_r \times 1}\), \(x(\cdot) \in \mathbb{C}^{N_x \times 1}\) is the intended signal from BS0, \(i_{agg}(\cdot, \cdot) \in \mathbb{C}^{N_r \times 1}\) is the aggregate other-cell interference and \(n(\cdot) \in \mathbb{C}^{N_r \times 1}\) is the Additive White Gaussian Noise (AWGN). The aggregate other-cell interference depends on \(r_0\), since the interfering BSs must lie outside the ball of radius \(r_0\) and centered at the origin. This originates from the shortest distance cell association criterion. More specifically: i) \(E\) is the BSs transmit-energy per transmission, which is equally split among the \(N_f\) antennas; ii) \(s_0(\tau) = \Theta^{(r, \tau)}(\eta_0; S_{(0)}^{(0)}) \in \mathbb{C}^{N_r \times 1}\), for \(t = 1, 2, \ldots, N_f\) and \(\tau = 1, 2, \ldots, N_s\), is the vector of space-time encoded symbols emitted by BS0, where \(\Theta^{(r, \tau)}(\cdot, \cdot)\) is the \(N_f \times N_r\) space-time encoding matrix, \(\eta_0\) is the vector of modulated information symbols and \(S_{(0)}^{(0)}\) is the side information available at BS0. In particular, \(M\) independent information symbols are transmitted in \(N_s\) time-slots, i.e., \(\eta_0\) is a \(M \times 1\) column-vector and \(\eta_0^{(m)} \in \mathbb{M}\) for \(m = 1, 2, \ldots, M\) with \(M\) denoting the set of modulated symbols. The generic \(M = \text{card}(\mathbb{M})\) symbols of \(\mathbb{M}\) are denoted by \(\mu_\ell \in \mathbb{M}\) for
\( x = 1, 2, \ldots, M \). A zero-mean and an average unit-energy constraints are assumed, i.e., 
\[
(1/M) \sum_{x=1}^{M} \mu_x = 0 \quad \text{and} \quad (1/M) \sum_{x=1}^{M} |\mu_x|^2 = 1,
\]
respectively. For example, they can be the means of either a Phase Shift Keying (PSK) or a Quadrature Amplitude Modulation (QAM) constellation diagram. The rate provided by (2.5.1) is 
\[
R = \left( \frac{M}{N_t} \right) \log_2 (M) \quad \text{bits per channel use (bpcu)};
\]
iii) \( H_0 \in \mathbb{C}^{N_r \times N_t} \) is the channel matrix of the BS-to-MT link, where 
\[
H_0^{(r,t)} = [H_0^{(r,t)}] \exp \left\{ j \arg \left\{ H_0^{(r,t)} \right\} \right\} \in \mathbb{C}^{N_r \times N_t} \quad \text{and} \quad \arg \left\{ H_0^{(r,t)} \right\} \sim U(0, 2\pi)
\]
for \( t = 1, 2, \ldots, N_t \) and \( r = 1, 2, \ldots, N_r \). Quasi-static fading is assumed in (2.5.1), which implies that \( H_0 \) is constant in the \( N_s \) times-slots of a frame, i.e., \( H_0(t) = H_0 \) for \( t = 1, 2, \ldots, N_s \), while it changes independently from one frame to another. The channel envelope \( |H_0^{(r,t)}| \) is assumed to follow a Rayleigh distribution[2.24]. The only exception is the SISO setup in Section 2.5.4.A, where \( |H_0^{(r,t)}| \) follows a Nakagami-\( m \) distribution with fading parameter \( m \) [2.24]; iv) \( b > 1 \) is the amplitude path-loss exponent; v) \( n_r(t) \sim \mathcal{CN}(0, \sigma_r^2) \) are independent and identically distributed (i.i.d.) RVs for \( r = 1, 2, \ldots, N_r \), \( t = 1, 2, \ldots, N_s \). Similar notation and assumptions are adopted for the interfering channels of \( i_{agg}(\cdot; \cdot) \). As for the other-cell interference model, (2.5.1) assumes the so-called isotropic scenario [2.10], where the \( N_r \) antennas at the MT are omnidirectional and are subject to the interference generated by all interfering BSs. Transmit- and receive-antennas are assumed to be co-located, hence the transmission distances \( r_0 \) and \( r_i \) for \( i \in \Psi^{(\text{agg})} \) are independent of the antennas inter-distances. Similar to the intended link, locations and channels of all interfering BSs are assumed not to change in the \( N_s \) times-slots of a frame. All channels are i.i.d. with mean square value 
\[
E \left\{ |H_0^{(r,t)}|^2 \right\} = E \left\{ |H^{(r,t)}|^2 \right\} = \Omega
\]
for \( t = 1, 2, \ldots, N_t \), \( r = 1, 2, \ldots, N_r \), \( i \in \Psi^{(\text{agg})} \). The signal model in (2.5.1) is sufficiently general to account for a large number of MIMO schemes, which are studied in Section 2.5.5.

2.5.3 Problem Formulation: Preliminaries

In this section, some definitions and enabling results for further analysis are introduced. The problem is formulated in a general manner, so that the MIMO arrangements analyzed in Section 2.5.4 turn out to be special cases of it. This provides a unified mathematical framework to the performance evaluation of MIMO cellular networks. As a consequence,
definitions and results presented in this and in the next section are formulated in terms of fundamental properties of the involved RVs, without linking them to a specific MIMO setup. The connection with MIMO cellular networks will become apparent in Section 2.5.4. In order to facilitate the reading, however, some remarks that link the general mathematical formulation used in this and in the next section to Section 2.5.4 are provided.

**Definition 2.5.1:** Let a complex RV \( X \). It is said to be spherically symmetric (or circularly symmetric or rotationally invariant) if its PDF, \( f_X(x) \), depends only on \( |X| \), i.e., \( f_X(x) = f_X(|x|) \) [2.53].

**Remark 2.5.1:** Let \( X = X^{(re)} + jX^{(im)} \) be a complex spherically symmetric RV. Then, the following properties hold [2.8]:

i) \( X = X \exp\{ j\phi \} \), where \( \phi \in [0, 2\pi) \) is an arbitrary constant,

ii) \( \Phi_X(\omega) = \Phi_X(|\omega|) \),

iii) \( \Phi_X^{(re)}(|\omega|) = \Phi_X^{(im)}(|\omega|) = \Phi_X(|\omega|) \),

iv) \( \Phi_X(|\omega|) = E_{X^{(re)}}\left[ \cos \left( |\omega| X^{(re)} \right) \right] = E_{X^{(im)}}\left[ \cos \left( |\omega| X^{(im)} \right) \right] \),

v) a linear combination of spherically symmetric RVs is still a spherically symmetric RV.

**Definition 2.5.2:** Let a complex RV \( X \). The RV \( X^{(GCG)} \) is said to be a Generalized Compound Gaussian (GCG) representation of \( X \) if the following equality in distribution holds:

\[
X \overset{d}{=} X^{(GCG)} = \sum_{q=1}^{\infty} B_q G_q
\]  

(2.5.2)

and: i) \( \{B_q\}^{\infty}_{q=1} \) are independent real RVs with \( M_{B_q}(s) = \exp\{ -s^2 \} \), ii) \( \{G_q\}^{\infty}_{q=1} \) are independent complex Gaussian RVs with distribution \( G_q \sim CN \left( 0, \sigma^2 \right) \), iii) \( \{B_q\}^{\infty}_{q=1} \) and \( \{G_q\}^{\infty}_{q=1} \) are independent RVs.

**Definition 2.5.3:** Let \( S_1 \) be the a priori information available at the MT about the received signal in (2.5.1), i.e., the side information. The Maximum-Likelihood (ML)-optimum [2.24]
demodulator based on (2.5.1) and $S_1$ is said to be $S_1$-optimum. If $S_1$ is independent of the other-cell interference, it is said to be interference-oblivious $S_1$-optimum.

**Remark 2.5.2:** As for the MIMO setups in Section 2.5.4, the a priori side information $S_1$ is usually a function of the channel matrix of the intended link, i.e., $S_1$ depends on $H_0$.

**Definition 2.5.4:** Let the received signal in (2.5.1). Let $z(\tau)$ be a short-hand for the $N_s \times N_t$ vectors $z(\tau)$ for $\tau = 1, 2, \ldots, N_\tau$. Let $\tilde{y}_0$ be the hypothesis of $y_0$ and $\Lambda_0(\tau) = \tilde{s}_0(\tau) - s_0(\tau) = \Theta(\tilde{y}_0; S_1^{(0)}) - \Theta(y_0; S_1^{(0)}) \in \mathbb{C}^{N_\tau \times 1}$. Let $S_1 = \sqrt{E/N_\tau}r_{0}^{-b}S_{\tau}$ be the side information at the MT, where $S_1$ is independent of $r_0$ and of the other-cell interference. Let $\Lambda(\cdot)$ be the decision metric of an interference-oblivious $S_1$-optimum demodulator based on (2.5.1). Let $\Lambda(\cdot)$ be written as:

\[
\Lambda(\Lambda_0(\tau)) = \sum_{\tau=1}^{N_\tau} \| \tilde{y}(\tau) - \bar{y}(\tau) \|^2
\]

\[
\propto (E/N_\tau)r_{0}^{-2b}D_1(\bar{S}_1, \Lambda_0(\tau)) + 2(E/N_\tau)r_{0}^{-2b}D_{LM}(\bar{S}_1, \Lambda_0(\tau))Re\{Y_{LM}\} + 2\sqrt{E/N_\tau}r_{0}^{-b}Re\{D_2(\bar{S}_1, \Lambda_0(\tau), \mathbf{n}(\tau))\} + 2\sqrt{E/N_\tau}r_{0}^{-b}Re\{D_3(\bar{S}_1, \Lambda_0(\tau), \mathbf{i}_{agg}(\tau; r_0))\}
\]

(2.5.3) where $\tilde{y}(\cdot)$ and $\bar{y}(\cdot)$ are demodulator- and modulator-dependent functions, respectively, $\bar{y}(\tau) = \bar{y}(\bar{S}_1, y(\tau)) \in \mathbb{C}^{N_\tau \times 1}$ is the post-processed received signal, $\tilde{y}(\tau) = \tilde{y}(S_1, s_0(\tau)) \in \mathbb{C}^{N_\tau \times 1}$ is the hypothesis at the receiver, $N$ is the vectors size that depends on the transmission scheme being considered, $D_1(\cdot, \cdot)$ and $D_{LM}(\cdot, \cdot)$ are positive real-valued functions, $D_2(\cdot, \cdot, \cdot)$ is, conditioning upon $\bar{S}_1$ and $\Lambda_0(\cdot)$, a complex Gaussian RV with zero mean, $D_3(\cdot, \cdot, \cdot)$ is a complex RV whose GCG representation is $D_3^{(GCG)}(\cdot, \cdot, \cdot)$ as follows:

\[
D_3(\bar{S}_1, \Lambda_0(\tau), \mathbf{i}_{agg}(\tau; r_0)) \overset{d}{=} D_3^{(GCG)}(\bar{S}_1, \Lambda_0(\tau), \mathbf{i}_{agg}(\tau; r_0)) = \sqrt{E/N_\tau} \sum_{q=1}^{N_\tau} (r_{0}^{-b+1/q}(p\lambda\tau)^{1/(2q)} \sqrt{B_{q}} G_{q})
\]

(2.5.4)
where $Y_{\text{IAI}} \sim \mathcal{CN}_{[0,\sigma^2_{\text{IAI}}]}(0,\sigma^2_{\text{IAI}})$ with $\sigma^2_{\text{IAI}} = \sigma^2_{\text{IAI}}(s_0(\tau))$, $M_{\text{B},s}(s) = \exp\{-s^q\}$, as well as $G \sim \mathcal{CN}(0,\sigma^2_{G})(S_1,\Lambda_0(\tau))$. If $D_{\text{IAI}}(\cdot) \neq 0$, the interference-oblivious $S_1$-optimum demodulator in (2.5.4), is said to be in a desired form if the following equalities hold:

$$
\frac{D_1^2(S_1,\Lambda_0(\tau))}{D_{\text{IAI}}^2(S_1,\Lambda_0(\tau))} = D_0(S_1,\Lambda_0(\tau))
$$

$$
\frac{D_2^2(S_1,\Lambda_0(\tau))}{E_{s(\tau)}\left\|D_2(S_1,\Lambda_0(\tau),n(\tau))\right\|^2} = \frac{D_0(S_1,\Lambda_0(\tau))}{N_0}
$$

(2.5.5)

where $D_0(\cdot)$ is a positive real-valued function and $\sigma^2_{G}$ is a non-negative constant related to $\sigma^2$ through the relation:

$$
\frac{D_1^2(S_1,\Lambda_0(\tau))}{\sigma^2_{G}(S_1,\Lambda_0(\tau))} = \frac{D_0(S_1,\Lambda_0(\tau))}{\sigma^2_{G}}
$$

(2.5.6)

If $D_{\text{IAI}}(\cdot) = 0$, the interference-oblivious $S_1$-optimum demodulator in (2.5.4) is said to be in a desired form if (2.5.5)

**Remark 2.5.3:** As for the MIMO setups in Section 2.5.5, the physical meaning of the addends in (2.5.4) is as follows: i) $D_1(\cdot,\cdot)$ is related to the serving BS; ii) $D_2(\cdot,\cdot,\cdot)$ is related to the AWGN at the receiver; iii) $D_3(\cdot,\cdot,\cdot)$ is related to the aggregate other-cell interference; iv) $D_{\text{IAI}}(\cdot)$ and $Y_{\text{IAI}}$ are related to the Inter-Antenna Interference (IAI) that may originate from coupling information symbols in space and time if $S_1$ is not sufficient enough for removing it at the receiver.

**Remark 2.5.4:** In (2.5.4), the proportionality symbol ($\propto$) is used because all terms independent of the hypothesis at the demodulator, i.e., $\Lambda_0(\cdot)$, are neglected. This is known not to introduce any sub-optimality in the definition of the demodulator and any approximations for its performance evaluation [2.24].
Remark 2.5.5: Let an interference-oblivious $S_1$-optimum demodulator formulated in the desired form in (2.5.4) and (2.5.5). It is said to be in a single-stream desired form if the decision metric in (2.5.4) can be written as follows:

$$\Lambda \left( \Lambda_0(\tau) \right) = \sum_{m=1}^{M} \Lambda_m \left( \Lambda_0(\tau) \right)$$  

(2.5.7)

where $\Lambda_m(\cdot)$ has the same structure as (2.5.3) except that it depends only on the $m$th information symbol, i.e., $D_{\chi}(\cdot) \mapsto D_{\chi,m}(\cdot)$ for $\chi = \{0,1,2,3,I\}$ and $Y_{I,m} \mapsto Y_{IAl,m}$. A similar notation holds for other symbols.

Remark 2.5.6: Demodulators formulated in the single-stream desired form in (2.5.7) allow the MT to demodulate the information vector $\eta_0$ in a symbol-by-symbol fashion without losing optimality. Demodulators formulated as in (2.5.3) need multi-stream algorithms because of the coupling of the $M$ information symbols.

In Section 2.5.6, performance metrics related to $S_1$-optimum demodulators are computed. In Section 2.5.5, it is shown that many MIMO detectors can be formulated as $S_1$-optimum demodulators.

2.5.4 Main Results

In this section, the main results of the paper are summarized. As anticipated in Section 2.5.3, the results are formulated in general terms based on the system model in Section 2.5.2 and subsequently they are linked to MIMO cellular networks, which are further investigated in Section 2.5.5.

Proposition 2.5.1: Let $\Psi$ and $\Psi^{(w)}$ be the PPPs of density $\lambda$ of available and interfering BSs, respectively. Let $p$ be the activity factors of the BSs. Let $b > 1$ be the amplitude path-loss exponent and $r_i > r_0$ for $i \in \Psi^{(w)}$ be the distances from the interfering BSs to the MT. Let $Z_{0,i}$ be i.i.d. spherically symmetric complex RVs for $i \in \Psi^{(w)}$ and let $Z_{0,i}^{(r)} = \Re \{ Z_{0,i} \}$ have zero mean and finite raw integer moments of any even order. Let $D_3(\cdot,\cdot,\cdot)$ in (2.5.3) as follows:

$$D_3 \left( S_1, \Lambda_0(\tau), i_{agg}(\tau;r_i) \right) = \sum_{i \in \Psi^{(w)}} \sqrt{E/N} r_i^{-b} Z_{0,i}$$  

(2.5.8)
The GCG representation of $D_3(\cdot,\cdot,\cdot)$ can be formulated as:

$$D_3\left(\tilde{S}_1,\Lambda_0(\tau),i_{agg}(\tau;\tau_0)\right) = D_3^{(GCG)}\left(\tilde{S}_1,\Lambda_0(\tau),i_{agg}(\tau;\tau_0)\right)$$

$$= \sqrt{E/N_s} \sum_{q=1}^{e_{SE}} \left(r_0^{(-b+1/q)} (p \lambda \pi)^{1/2} B_q G_q \right)$$

where $M_{B_q}(s) = \exp\{-s^q\}$ and $G_q \sim \mathcal{CN}\left(0,\sigma^2_{\tilde{G}_q}\left(\tilde{S}_1,\Lambda_0(\tau)\right)\right)$ with:

$$\sigma^2_{\tilde{G}_q}\left(\tilde{S}_1,\Lambda_0(\tau)\right) = \left\{(-1)^q \frac{(-1/b)_q}{(1/2)_q (1-1/b)_q} \frac{1}{q!} E\left(\left(Z_{0,i}^{(re)}\right)^{2q}\right)\right\}^{1/q}$$

(2.5.9)

The proof is available in [2.55].

**Remark 2.5.7:** As for the MIMO setups in Section 2.5.5, $D_3(\cdot,\cdot,\cdot)$ in (2.5.8) represents the aggregate other-cell interference at the output of the demodulator and $Z_{0,i}$ for $i \in \Psi^{(\omega)}$ is the contribution originating from each interfering BS.

**Lemma 2.5.1:** Let $Z_{0,i}$ be a complex RV defined as follows:

$$Z_{0,i} = \sum_{r=1}^{N_r} \sum_{t=1}^{N_t} \sum_{r=1}^{N_r} H_i^{(r,t)} s_i^{(t)}(\tau) U_0^{(r)}(\tau)$$

(2.5.10)

where $H_i^{(r,t)}$ are i.i.d. complex Gaussian RVs, i.e., $H_i^{(r,t)} \sim \mathcal{CN}\left(0,\Omega\right)$ for $r=1,2,\ldots,N_r$, $t=1,2,\ldots,N_t$, $s_i^{(t)}(\tau)$ and $U_0^{(r)}(\tau)$ are $N_t \times 1$ and $N_r \times 1$ complex random vectors, respectively, for $\tau=1,2,\ldots,N_r$. Let $Z_{0,i}^{(re)} = \text{Re}\{Z_{0,i}\}$. The raw integer moments of any even order of $Z_{0,i}^{(re)}$ with respect to $H_i$ and $s_i^{(t)}(\tau)$ are:

$$E\left(Z_{0,i}^{(re)}\right)^{2q} = E_{s_i^{(t)}(\tau)}\left(E_{H_i}\left(Z_{0,i}^{(re)}\right)^{2q}\right) = \sqrt{\pi} \frac{\Gamma(1/2-k)}{(1/2-k)} (-1)^q \Omega^{q} E_{s_i^{(t)}(\tau)}\left(\sum_{r=1}^{N_r} \sum_{t=1}^{N_t} s_i^{(t)}(\tau) U_0^{(r)}(\tau)^2\right)$$

(2.5.11)

**Remark 2.5.8:** As for the MIMO setups in Section 2.5.5, $Z_{0,i}$ in (2.5.10) depends on the channel gains, $H_i^{(r,t)}$, and on the information symbols, $s_i^{(t)}(\cdot)$, of the interfering BSs, as well as on the information symbols, $U_0^{(\cdot)}(\cdot)$, of the serving BS. Since $H_i^{(r,t)}$ are complex Gaussian RVs, their phase is uniformly distributed and, thus, from Definition 2.5.1, they are spherically...
symmetric RVs. Since \( Z_{0,i} \) is the linear combination of \( H_i^{(r,i)} \), according to Remark 2.5.1, it is spherically symmetric as well.

**Lemma 2.5.2:** Let \( \sigma_{\mathcal{Q}_0}^2(\cdot,\cdot) \) in (2.5.9) with \( E\left\{\left(Z_{0,i}^{(r,i)}\right)^{2q}\right\} \) formulated as in (2.5.11). Let a function \( \phi(\cdot) \) such that:

\[
\sum_{r=1}^{N_r} \sum_{r=1}^{N_r} \sum_{i=1}^{N_i} s_i^{(r)}(\tau) U_0^{(r)}(\tau) = \phi(\cdot) \sum_{r=1}^{N_r} \sum_{r=1}^{N_r} U_0^{(r)}(\tau)^2
\]

(2.5.12)

Let \( \sigma_{\mathcal{Q}_0}^2 \) satisfy the equality in (2.5.6) with

\[
D_i^2 \left( S_i, \Lambda_0(\tau) \right)/D_0 \left( S_i, \Lambda_0(\tau) \right) = \sum_{r=1}^{N_r} \sum_{r=1}^{N_r} U_0^{(r)}(\tau)^2.
\]

Let \( Q(X) = \sum_{q=1}^{+\infty} \left( X \sigma_{\mathcal{Q}_0}^2 \right)^q \). For every \( X > 0 \), the following identity holds:

\[
Q(X) = E_{s_i(\tau)} \left\{ F_1 \left( -\frac{1}{b}; 1 - \frac{1}{b} \phi(s_i(\tau)) \Omega X \right) \right\} - 1
\]

(2.5.13)

**Remark 2.5.9:** If \( N_s = 1 \), the equality in (2.5.12) is always satisfied with

\[
\phi(s_i(1)) = \sum_{r=1}^{N_r} s_i^{(r)}(1)^2,
\]

i.e.,

\[
\sum_{r=1}^{N_r} \sum_{r=1}^{N_r} U_0^{(r)}(1)^2 = \left( \sum_{r=1}^{N_r} s_i^{(r)}(1)^2 \right) \left( \sum_{r=1}^{N_r} U_0^{(r)}(1)^2 \right).
\]

AEP_{\mathcal{W}}(\mathbf{u}_0, \mathbf{u}_b) = E_{\mathcal{W}_0} \left\{ E_{\left\{B_{\mathcal{W}}\right\}_{q=1}} \left\{ P_{\mathcal{W}} \left( \sigma_{\mathcal{W}}^2, \mathbf{u}_0, \left\{B_{\mathcal{W}}\right\}_{q=1}; 2 \pi / 2, \pi / 2 \right) \right\} \right\}.

(2.5.20)

**Lemma 2.5.2:** Let a complex RV \( Z_{0,i} = h_i s_i h_0 i \delta_0 \), where \( |h_i|^2 \sim G(m, \Omega) \), \( \arg(h_i) \sim U(0, 2\pi) \), while \( s_i, h_0 \) and \( i \delta_0 \) are complex random numbers. Let \( Z_{0,i}^{(r,i)} = \text{Re}\left\{Z_{0,i}\right\} \). The raw integer moments of any even order of \( Z_{0,i}^{(r,i)} \) with respect to \( h_i \) and \( s_i \) are as follows:

\[
E \left( \left( Z_{0,i}^{(r,i)} \right)^{2q} \right) = E_{s_i} \left\{ E_{h_i} \left( \left( Z_{0,i}^{(r,i)} \right)^{2q} \right) \right\} = \left| h_0 \right|^{2q} \delta_0^{2q} \left[ \frac{\Gamma(m + q) \Gamma(q + 1)}{m^q \Gamma(m) \sqrt{\pi \Gamma(q + 1)} \Omega^q} \right] E_{s_i} \left\{ \left| s_i \right|^{2q} \right\}
\]

(2.5.14)

**Remark 2.5.10:** The formulation of \( Z_{0,i} \) in Lemma 2.5.2 is used to study the SISO setup of Section 2.5.5 I. Since the phase of \( h_i \) is uniformly distributed, \( Z_{0,i} \) is spherically symmetric, according to Remark 2.5.1, as well.
Lemma 2.5.3: Let $\sigma_{\Delta_0}^2(\cdot,\cdot)$ in (2.5.9) with $E\left\{Z_{0,\Delta_0}^{(\cdot,\cdot),q}\right\}$ formulated as in (2.5.13). Let $\sigma_{\Delta_0}^2$ satisfy the equality in (2.5.6) with $D_0^2(\bar{S}_i,\Delta_0(\tau))/D_0(\bar{S}_i,\Delta_0(\tau)) = |h_0|^2|\delta_0|^2$. Let $Q(X) = \sum_{q=1}^{\infty} X \sigma_{\Delta_0}^2 \right)^q$. For every $X > 0$, the following identity holds:

$$Q(X) = E_x \left\{ 2F_2 \left(-\frac{1}{b},m;1;\frac{1}{b};1;\frac{1}{2}\Omega X \right) \right\} - 1 \quad (2.5.15)$$

Lemma 2.5.4: Let a bi-dimensional modulation scheme with equi-probable symbols, which is identified by the quadruplets of parameters $(\alpha_1,\beta_1,\gamma_1,\delta_1)$ and $(\alpha_2,\beta_2,\gamma_2,\delta_2)$ [2.10] Let an interference-oblivious $S_1$-optimum demodulator formulated in the single-stream desired form of (2.5.7), as:

$$\hat{n}_b^{(m)} = \arg\min_{\hat{n}_b^{(m)} \in \mathbb{M}} \left\{ A_m(\Delta_0(\tau)) \right\} \quad (2.5.16)$$

Let $D_{0,m}(\bar{S}_t,\Delta_0(\tau)) = \bar{D}_o(\bar{S}_t)\bar{D}_{0,m}(\Delta_0(\tau))$ with $E_{\Delta_0(\tau)}{D_{0,m}(\Delta_0(\tau))} = 2$. The Average Symbol Error Probability (ASEP) of (2.5.1) i.e., $\text{ASEP}_m = \Pr\{\hat{n}_b^{(m)} \neq n_b^{(m)}\}$ can be formulated as in (2.15), where:

$$P_E \left(\sigma_{\Delta_0}^2, r_0, \left\{B_q\right\}_{q=1}^{\infty};\alpha,\beta,\gamma,\delta\right) = \frac{1}{\pi} \int_0^\infty M_{\Delta_0} \left(\frac{\beta \sin^2(\gamma)}{2\sin^2(\omega)} \text{SINR} \left(\sigma_{\Delta_0}^2, r_0, \left\{B_q\right\}_{q=1}^{\infty} \right) \right) d\omega \quad (2.5.17)$$

with $M_{\Delta_0}(s) = E_{\Delta_0} \left\{ \exp\left\{-s\bar{D}_o(\bar{S}_t)\right\} \right\}$ and $\text{SINR}(\cdot,\cdot,\cdot)$ is defined in (2.5.18) and (2.5.19)

$$M_{\text{SINR}}(s) = E_{\Delta_0} \left\{ E_{\left|\Delta_0\right|}^{\infty} \left\{ M_0 \left( s \text{SINR} \left(\sigma_{\Delta_0}^2, r_0, \left\{B_q\right\}_{q=1}^{\infty} \right) \right) \right\} \right\}$$

$$= 1 - m_0 \Omega_0 (E/L_N) \pi \lambda s$$

$$x \int_0^{\infty} \int_0^{\infty} F_1 \left(m_0 + 1;2;\Omega_0 (E/L_N) s \right) P_{\text{M}_1} \left( (E/L_N) y \right) \exp\left(-N_0 x^2 y\right) \exp\left(-\pi \lambda Q \left( (E/L_N) y \right) x \right) dx dy$$

$$\text{SINR} \left(\sigma_{\Delta_0}^2, r_0, \left\{B_q\right\}_{q=1}^{\infty} \right) = \frac{E}{N_t} \left(\frac{r_0 - b}{N_t} \right)^2 N_0 + \frac{E}{N_t} \left(\frac{r_0 - b}{N_t} \right)^2 \sigma_{\Delta_0}^2 + \frac{E}{N_t} \sum_{q=1}^{\infty} \left(\frac{r_0 - b}{N_t} \right)^2 \left( p \lambda \pi \right)^{1/q} B_q \sigma_{\Delta_0}^2 \right)^{-1} \quad (2.5.19)$$

The mathematical formulation of the ASEP is possible thanks to the EID-based representation of the aggregate other-cell interference, i.e., $D_3(\cdot,\cdot,\cdot)$. In fact, by conditioning upon $\sigma_{\Delta_0}^2$, $r_0$, and $\left\{B_q\right\}_{q=1}^{\infty}$, the decision metric of the demodulator in (2.5.15) boils down to that of an equivalent demodulator in AWGN. As a consequence, the widely adopted
mathematical formulation of the ASEP of bi-dimensional modulations can be used. As a first step, in fact, only the randomness of the AWGN and of the complex Gaussian RVs \(G_q\) is taken into account. The conditioning with respect to \(\sigma_{1A}^2, r_0,\) and \(\{B_q\}_q\) is removed subsequently.

The constraint \(E_{\Lambda_0(\tau)}\left\{D_{0,m}(\Lambda_0(\tau))\right\}=2\) represents a normalization factor that can be understood, for specific MIMO setups, by direct inspection of Section 2.5.5. From Section 2.5.5, in particular, \(D_{0,m}(\Lambda_0(\tau))=|\Lambda_0(\tau)|^2\), which implies \(E_{\Lambda_0(\tau)}\left\{D_{0,m}(\Lambda_0(\tau))\right\}=2\) as a result of the zero-mean and average unit-energy constraints assumed for the constellation diagram.

**Lemma 2.5.5**: Let an interference-oblivious \(S_1\)-optimum demodulator formulated in the desired form of (2.5.3), as:

\[
\hat{\eta}_0 = \arg\min_{\hat{\eta}_0 \in M^{\eta_0}} \left\{A(\Lambda_0(\tau))\right\}
\]  

(2.5.20)

The Average Pairwise Frame Error Probability (APEP\(_F\)), which is defined as \(\text{APEP}_F(\eta_0 \rightarrow \hat{\eta}_0) = \Pr\left\{A(\Lambda_0(\tau)) < 0\right\}\), can be formulated as

\[
P_{\text{EF}}(\sigma_{1A}^2, r_0, \{B_q\}_q^{+\infty}; \alpha, \beta, \gamma) = \frac{1}{\pi} \int_0^\pi M_{D_0} \left( \beta \frac{\sin^2(\gamma)}{2\sin^2(\omega)} \text{SINR} \left( \sigma_{1A}^2, r_0, \{B_q\}_q^{+\infty} \right) \right) d\omega(2.5.21)
\]

with \(M_{D_0}(s) = E_{S_1} \left\{ \exp \left\{ -sD_0(\hat{S}_1, \Lambda_0(\tau)) \right\} \right\}\).

**Remark 2.5.11**: From APEP\(_F\), the Average Frame Error Probability (AFEP) of bi-dimensional modulations can be obtained by using the Nearest Neighbor (NN) approximation. The AFEP can be calculated by considering only the pairs of transmitted (\(\eta_0\)) and hypothesis (\(\hat{\eta}_0\)) vectors that differ in a single entry. Assuming that the \(M\) possibilities when this condition holds are equiprobable, the ASEP follows from the AFEP and can be formulated as \(\text{ASEP} = \text{AFEP} / M\).
Theorem 2.5.1: Let \( M_0(s) = (1 + s\Omega_0)^{-m_0} \), \( f_0(\xi) = 2\pi\lambda\xi \exp\{-\pi\lambda\xi^2\} \) and \( M_{B_q}(s) = \exp\{-s^q\} \) for every integer \( q \). Let the SINR(\cdot,\cdot) defined in (2.5.19). Its MGF can be formulated as shown in (2.5.18), where \( P_{\text{IAI}}(\xi) = E_{f_0(\tau)}\{\exp\{-\sigma_{\text{IAI}}^2(s_0(\tau))\xi\}\} \), \( \tilde{Q}(\xi) = \mu Q(\xi) + 1 \) and \( Q(\xi) = \sum_{q=1}^{+\infty}(\xi^q\sigma_{G_q}^2)^q \).

Corollary 2.5.1: Let the SINR(\cdot,\cdot) in (2.5.18) with \( E/N_0 \to +\infty \), i.e., an interference-limited regime is considered. Then, \( M_{\text{SINR}}(\cdot) \) in (2.5.19) simplifies as follows:
\[
M_{\text{SINR}}(s) \big|_{E/N_0 \to +\infty} = M_{\text{SINR}}^{(s)}(s) = 1 - m_0\Omega_0s \int_0^{+\infty} F_1(m_0 + 1; 2; -\Omega_0sz) P_{\text{IAI}}(z) dz \tag{2.5.22}
\]

Theorem 2.5.2: Let \( M_0(s) = (1 + s\Omega_0)^{-m_0} \), \( f_0(\xi) = 2\pi\lambda\xi \exp\{-\pi\lambda\xi^2\} \), \( M_{B_q}(s) = \exp\{-s^q\} \) for every integer \( q \). Let the SINR(\cdot,\cdot) and the error probability integral in (2.5.19) and (2.5.17), respectively. The identity in (2.5.18) holds, where \( \kappa = \beta\sin^2(\gamma)/2 \), \( P_{\text{IAI}}(\xi) = E_{f_0(\tau)}\{\exp\{-\sigma_{\text{IAI}}^2(s_0(\tau))\xi\}\} \), \( \tilde{Q}(\xi) = \mu Q(\xi) + 1 \), \( Q(\xi) = \sum_{q=1}^{+\infty}(\xi^q\sigma_{G_q}^2)^q \) and \( T(\cdot;\cdot;\cdot) \) is defined in [2.55].

2.5.5 Application to MIMO Cellular Networks
In this section, various MIMO arrangements are studied and it is proved that their error probability performance can be formulated as in Section 2.5.4. By direct inspection of Theorem 2.5.1 and Theorem 2.5.2, it is apparent that the error probability integrals in depend only on three parameters, i.e., \( m_0, \Omega_0 \) and \( Q(\cdot) \). In the following sub-sections, such a triplet of parameters is computed for relevant MIMO schemes. It is worth mentioning that for some MIMO setups the proposed mathematical framework provides only approximated expressions of the error probability, as explained in the sequel. A summary of the triplet \((m_0,\Omega_0,Q(\cdot))\) is provided in [2.55] for all MIMO schemes analyzed in the present report. Also, [2.55] highlights when the framework is exact or approximated.

For all analyzed MIMO transmission schemes, the following procedure is applied:
1) From the signal model in (2.5.1) and the demodulator in (2.5.3), the functions $D_1(\cdot, \cdot)$, $D_2(\cdot, \cdot, \cdot)$ and $D_3(\cdot, \cdot, \cdot)$ are computed. If $\sigma^2_{I_{AI}} \neq 0$, $D_{I_{AI}}(\cdot, \cdot, \cdot)$ is computed as well. Otherwise, the MIMO scheme is IAI-free.

2) The function $D_0(\cdot, \cdot)$ is computed such that the constraints in (2.5.4) and (2.5.6) are all satisfied. The difference between exact and approximated results may emerge at this step: if all constraints are satisfied with equality, the mathematical formulation is exact. Otherwise, it is an approximation.

3) By letting $M_0(s) = M_{D_0}(s)$ for the single-stream demodulator and $M_0(s) = M_{D_0}(s)$ for the multi-stream demodulator, the parameters $m_0$ and $\Omega_0$ are computed from $D_0(\cdot, \cdot)$ such that $M_0(s) = (1 + s\Omega_0)^{-m_0}$ holds.

### 2.5.5 Single-Input-Single-Output Transmission over Nakagami-m Fading

Let a SISO transmission scheme and a Nakagami-m fading channel model. Thus, $N_s = N_r = N_t = 1$, $M = 1$, $s_0^{(1)}(1) = \eta_0^{(1)}$, $s_0^{(1)}(1) = \eta_0^{(1)}$ for $i \in \Psi^{(s)}$, $|H_0^{(1)}|^2 \sim G(m, \Omega)$ and $|H_j^{(1)}|^2 \sim G(m, \Omega)$ for $i \in \Psi^{(s)}$. Let the interference-oblivious demodulator in (2.5.7) with $\bar{S}_i = H_0^{(1)}$, $\bar{y}(1) = \bar{\psi}(S_0, y(1)) = y(1)$, $\bar{y}(1) = \bar{\psi}(S_1, \bar{s}_0(1)) = \sqrt{\text{E}_0^{-b}} H_0^{(1)} \bar{\eta}_0^{(1)}$ and $N = 1$.

Since a single symbol is transmitted, the demodulator is IAI-free and $\sigma^2_{I_{AI}} = 0$. By inserting (2.5.1) in (2.5.3), we obtain $\Lambda_0^{(1)}(1) = \bar{\eta}_0^{(1)} - \eta_0^{(1)}$, $D_{2,1}(\bar{S}_1, \Lambda_0(\tau)) = |H_0^{(1)}|^2 |\Lambda_0^{(1)}(1)|^2$, $D_{2,1}(\bar{S}_1, \Lambda_0, \mathbf{n}(\tau)) = (H_0^{(1)} \Lambda_0^{(1)}(1))^* \mathbf{n}^{(1)}(1)$ and $D_{3,1}(\cdot, \cdot, \cdot)$ formulated as in (2.5.8) with $Z_{0,i} = (H_0^{(1)} \Lambda_0^{(1)}(1))^* H_0^{(1)} s_i^{(1)}(1)$. From (2.5.4), $D_{0,1}(\bar{S}_1, \Lambda_0(\tau)) = |H_0^{(1)}|^2 |\Lambda_0^{(1)}(1)|^2$. Thus, $\overline{D}_0(\bar{S}_1) = |H_0^{(1)}|^2 \sim G(m, \Omega)$ and $(m_0, \Omega_0) = (m, \Omega / m)$. $Q(\cdot)$ follows from (2.5.14).

### 2.5.5 Spatial Multiplexing MIMO Transmission over Rayleigh Fading -- Optimal Demodulation

Let a spatial multiplexing MIMO transmission scheme and a Rayleigh fading channel model. Thus, $N_s = 1$, $M = N_t$, $s_{0}(1) = \eta_{0}$, $s_i(1) = \eta_i$ for $i \in \Psi^{(s)}$, $|H_0^{(r, s)}|^2 \sim G(1, \Omega)$ and
\[ |H^{(r,j)}_t|^2 \sim G(1, \Omega) \text{ for } t = 1, 2, \ldots, N_r \text{, } r = 1, 2, \ldots, N_r \text{ and } i \in \Psi^{(w)} \]. Let the interference-oblivious demodulator in (2.5.3) with \( \tilde{S}_1 = H_0 \), \( y(1) = \tilde{\psi}(S_1, y(1)) = y(1) \), \( \bar{y}(1) = \tilde{\psi}(S_1, \bar{s}_0(1)) = \sqrt{E/N}r_NH_0\tilde{n}_0 \) and \( N = N_r \). With this choice, the multi-stream demodulator is IAI-free and \( \sigma_{\text{IAI}}^2 = 0 \), as well as \( \Lambda_0(1) = \tilde{n}_0 - \eta_0 \), \( D_1(\tilde{S}_1, \Lambda_0(\tau)) = \sum_{r=1}^{N_r} \sum_{t=1}^{N} H^{(r,2)}_t \Lambda_0^{(l)}(1) \), \( D_2(\tilde{S}_1, \Lambda_0(\tau), n(\tau)) = \sum_{r=1}^{N_r} \sum_{t=1}^{N} \left( H^{(r,1)}_0 \Lambda_0^{(l)}(1) \right)^* n^{(r)}(1) \), and \( D_3(\tilde{\cdots}) \) as in (2.5.8) with \( Z_{0,i} = \sum_{r=1}^{N_r} \sum_{t=1}^{N} H^{(r,i)}_t \left( s_0^{(l)}(1) U_0^{(r)}(1) \right) \) and \( U_0^{(r)}(1) = \sum_{r=1}^{N_r} \left( H^{(r,2)}_0 \Lambda_0^{(l)}(1) \right)^* . \) From (2.5.4), \( D_0(\tilde{S}_1, \Lambda_0(\tau)) = \sum_{r=1}^{N_r} \sum_{t=1}^{N} H^{(r,1)}_0 \left( \Lambda_0^{(l)}(1) \right)^2 \). Since \( \sum_{r=1}^{N_r} H^{(r,1)}_0 \Lambda_0^{(l)}(1) \sim CN \left[ 0, \Omega \sum_{r=1}^{N_r} \left( \Lambda_0^{(l)}(1) \right)^2 \right] \), then \( D_0(\tilde{S}_1, \Lambda_0(\tau)) \sim G \left[ N_r, N_r, \Omega \sum_{r=1}^{N_r} \left( \Lambda_0^{(l)}(1) \right)^2 \right] \) and \( (m_0, \Omega_0) = \left( N_r, \Omega \sum_{r=1}^{N_r} \left( \Lambda_0^{(l)}(1) \right)^2 \right) \). Finally, \( Q(\cdot) \) follows from (2.5.12) with \( \phi(s_i(1)) = \sum_{t=1}^{N} |s_i^{(l)}(1)|^2 \).

### 2.5.5 Single-Input-Multiple-Output (SIMO) Transmission over Rayleigh Fading

Let a SIMO transmission scheme and a Rayleigh fading channel model. Thus, \( N_i = N_s = 1 \), \( \sum_{r=1}^{N_r} H^{(r,1)}_0 \sim G(1, \Omega) \) and \( \sum_{r=1}^{N_r} H^{(r,2)}_t \sim G(1, \Omega) \) for \( r = 1, 2, \ldots, N_r \) and \( i \in \Psi^{(w)} \). Let the interference-oblivious demodulator in (2.5.7) with \( \tilde{S}_1 = H_0^{(r,j)} \) for \( r = 1, 2, \ldots, N_r \), \( y(1) = \tilde{\psi}(S_1, y(1)) = y(1) \), \( \bar{y}^{(r)}(1) = \tilde{\psi}(S_1, \bar{s}_0(1)) = \sqrt{E/r_b} H^{(r,1)}_0 \bar{n}_0 \) for \( r = 1, 2, \ldots, N_r \) and \( N = N_r \). Since a single symbol is transmitted, the demodulator is IAI-free and \( \sigma_{\text{IAI}}^2 = 0 \). By inserting (2.5.1) in (2.5.3), we obtain \( \Lambda_0^{(l)}(1) = \eta_0^{(l)} - s_0^{(l)} \), \( D_{1,1}(\tilde{S}_1, \Lambda_0(\tau)) = \left| \Lambda_0^{(l)}(1) \right|^2 \sum_{r=1}^{N_r} \left| H^{(r,3)}_0 \right|^2 \), \( D_{2,1}(\tilde{S}_1, \Lambda_0(\tau), n(\tau)) = \left( \Lambda_0^{(l)}(1) \right)^* \sum_{r=1}^{N_r} \left( H^{(r,1)}_0 \right)^* n^{(r)}(1) \) and \( D_{3,1}(\tilde{\cdots}) \) formulated as in (2.5.8) with \( Z_{0,i} = \sum_{r=1}^{N_r} H^{(r,2)}_t s_0^{(l)}(1) U_0^{(r)}(1) \) and \( U_0^{(r)}(1) = \left( H_0^{(r,3)} \Lambda_0^{(l)}(1) \right)^* \). From (2.5.4), \( D_{0,1}(\tilde{S}_1, \Lambda_0(\tau)) \)
\[ \Delta_i^{(1)}(1) = \sum_{r=0}^{N_r} |\mathbf{H}_0^{(r,1)}|^2. \] Thus, \( \bar{D}_0(\bar{S}_1) = \sum_{r=1}^{N_r} |\mathbf{H}_0^{(r,1)}|^2 \sim G(N_r, N_s, \Omega) \) and \((m_0, \Omega_0) = (N_s, \Omega)\).

Finally, \( Q(\cdot) \) follows from (2.5.12) with \( \phi(s_i(1)) = |s_i^{(1)}(1)|^2 \).

### 2.5.5 Orthogonal Space-Time Block Coding (OSTBC) Transmission over Rayleigh Fading

Let an OSTBC MIMO transmission scheme and a Rayleigh fading channel model. Based on [2.56], generalized complex orthogonal designs of size \( N_s \) are considered. Thus, \( M / N_s \leq 1 \) and the \( N_s \times N_s \) space-time encoding matrix \( \Theta(\eta_0) = S_1^{(m)} = \Theta(\eta_0) \) satisfies the property \( \Theta^H(\eta_0) \Theta(\eta_0) = \mathbf{D}_\Theta(\eta_0) \), where \( \mathbf{D}_\Theta^{(i,j)}(\eta_0) = \sum_{m=1}^{M} p_{ij}^{(m)} |\eta_0^{(m)}|^2 \) for \( i = j = t = 1,2,\ldots,N_s \), \( \mathbf{D}_\Theta^{(i,j)}(\eta_0) = 0 \) for \( i \neq j = t = 1,2,\ldots,N_s \), and \( p_{ij}^{(m)} \) are strictly positive numbers for \( t = 1,2,\ldots,N_s \), and \( m = 1,2,\ldots,M \). For example, \( \bar{p} = p_{ij}^{(m)} \) for \( t = 1,2,\ldots,N_s \) and \( m = 1,2,\ldots,M \) with \( \bar{p} = 2 \) and \( \bar{p} = 1 \) if the matrices in [2.56] and in [2.56] are considered, respectively. As for the channels, \( |\mathbf{H}_0^{(r,1)}|^2 \sim G(1, \Omega) \) and \( |\mathbf{H}_r^{(r,1)}|^2 \sim G(1, \Omega) \) for \( t = 1,2,\ldots,N_s \), \( r = 1,2,\ldots,N_r \) and \( i \in \Psi^{(m)} \). Let the interference-oblivious demodulator in (2.5.7) with \( \bar{S}_1 = \mathbf{H}_0 \), \( \bar{y}(\tau) = \psi(S_1, y(\tau)) = y(\tau), \bar{y}(\tau) = \psi(S_1, \bar{S}_0(\tau)) = \sqrt{E / N_s r_0} \Theta(\eta_0) \) and \( N = N_r \). With this choice, the multi-stream demodulator is IAI-free and the multi-stream demodulator can be re-written in terms of the single-stream demodulator in (2.5.1) by exploiting the properties of \( \Theta(\cdot) \). In particular, let \( \Delta_i^{(\cdot)} \) for \( i \in \Psi \) be formulated as follows \( (\tau = 1,2,\ldots,N_s) \):

\[ \Delta_i^{(\tau)}(\tau) = \Theta^{(\cdot,\tau)}(\bar{\eta}_0) - \Theta^{(\cdot,\tau)}(\eta_0) = \sum_{m=1}^{M} a_{ij}^{(\tau)}(\tau)(\bar{\eta}_0^{(m)} - \eta_0^{(m)}) + \sum_{m=1}^{M} b_{ij}^{(\tau)}(\tau)(\bar{\eta}_0^{(m)} - \eta_0^{(m)})^* \] (2.5.23)

where \( a(\cdot) \) and \( b(\cdot) \) are \( N_s \times 1 \) complex vectors for \( \tau = 1,2,\ldots,N_s \), which depend on the space-time encoding matrix \( \Theta(\cdot) \). Then, with the aid of the property [2.56] as follows (it holds for \( i \in \Psi \)):

\[ \sum_{r=1}^{N_r} \sum_{t=1}^{N_s} \sum_{r=1}^{N_r} |\mathbf{H}_r^{(r,\tau)}\Delta_i^{(\tau)}(\tau)|^2 = \sum_{r=1}^{N_r} \sum_{t=1}^{N_s} |\mathbf{H}_r^{(r,\tau)}|^2 \left( \sum_{m=1}^{M} |\eta_0^{(m)}|^2 \right)^2 \] (2.5.24)

we obtain, for \( m = 1,2,\ldots,M \), the identities in (2.5.25) and \( \bar{D}_{3,m}(\cdot,\cdot,\cdot) \) can be formulated with the definitions given in (2.5.26)
\[
D_{1,m}(\mathbf{S}_t, \mathbf{A}_0(\tau)) = \left| \tilde{\eta}_0^{(m)} - \eta_0^{(m)} \right|^2 \sum_{i=1}^{N_r} \left| \mathbf{H}_0^{(r,i)} \right|^2 D_{2,m}(\mathbf{S}_t, \mathbf{A}_0(\tau), \mathbf{n}(\tau))
\]

\[= \sum_{r=1}^{N_r} \sum_{i=1}^{N_t} \left( \mathbf{u}^{(r)}(\tau) \left( \mathbf{a}^{(r)}(\tau) (\tilde{\eta}_0^{(m)} - \eta_0^{(m)}) + \mathbf{b}^{(r)}(\tau) (\tilde{\eta}_0^{(m)} - \eta_0^{(m)})^* \right) \right) \]

\[Z_{0,i} = \sum_{r=1}^{N_r} \sum_{i=1}^{N_t} \left( \mathbf{H}_0^{(r,i)} s_i^{(u)}(\tau) \right) U_0^{(r)} = \sum_{r=1}^{N_r} \sum_{a=1}^{N_a} \mathbf{H}_0^{(r,a)} s_a^{(u)}(\tau) U_0^{(r)}(\tau)
\]

\[U_0^{(r)} = \sum_{i=1}^{N_t} \mathbf{H}_0^{(r,i)} \left( \mathbf{a}^{(r)}(\tau) (\tilde{\eta}_0^{(m)} - \eta_0^{(m)}) + \mathbf{b}^{(r)}(\tau) (\tilde{\eta}_0^{(m)} - \eta_0^{(m)})^* \right)
\]

It is apparent that \(Z_{0,i}\) in (2.5.26) is formulated as shown in (2.5.10) of Lemma 1. To proceed with the analysis, it is important to understand whether the equality in (2.5.13) is satisfied for arbitrary generalized complex orthogonal designs. This issue is addressed in the following.

**Definition 2.5.5:** Let a generalized complex orthogonal design \(\Theta\), according to [2.56]. Let \(Z_{0,i}\) in (2.5.26). \(\Theta\) is said to be interference-orthogonal if the equality in (2.5.13) is satisfied for \(\phi(\mathbf{s}_i(\tau)) = \sum_{m=1}^{M} \left| \eta_i^{(m)} \right|^2\).

**Proposition 2.5.2:** The generalized complex orthogonal designs in [2.56] are interference-orthogonal, while those in [2.56] are not interference-orthogonal.

The reason why some generalized complex orthogonal designs are not interference-orthogonal is due to the quasi-static assumption for the other-cell interference. This implies, in fact, that the terms \(i^{(r)}(\tau) = \sum_{a=1}^{N_a} \mathbf{H}_0^{(r,a)} s_a^{(u)}(\tau)\) in (2.5.26) are not independent, since they originate from interfering BSs belonging to the same PPP. Comparing \(D_{2,m}(\cdot, \cdot, \cdot)\) in (2.5.25) with \(Z_{0,i}\) in (2.5.26), we note that this does not occur for \(D_{2,m}(\cdot, \cdot, \cdot)\), since the noise terms \(\mathbf{n}^{(r)}(\tau)\) are independent for \(r=1, 2, \ldots, N_r\), \(\tau=1, 2, \ldots, N_s\). The generalized complex orthogonal designs in [2.56] are designed based on the independence property of the AWGN. Hence, some code constructions may not satisfy the interference-orthogonal property that originates from the partial correlation of the interference across the receive-antennas and the time-slots.
The proposed mathematical approach is applicable to interference-orthogonal generalized complex orthogonal designs. It can be applied to generalized complex orthogonal designs that are not interference-orthogonal, by assuming that the equality in (2.5.13) holds true. In this latter case, the framework is no longer exact, but it is an approximation. In Section 2.5.6, it is shown that it is accurate enough for typical MIMO setups though.

If $\Theta$ is interference-orthogonal, the equalities in (2.5.4) are satisfied and we obtain
\[
D_{0,m}(\bar{S}_1, \Delta_0(\tau)) = \left| \eta_0^{(m)} - \eta_0^{(m)} \right|^2 + \sum_{t=1}^{N_f} \sum_{r=1}^{N_r} \left| p_r^{(r)} \right|^2 \mathbb{E} \left[ H_0^{(r,t)} \right]^2 \quad \text{and} \quad D_{0,m}(\bar{S}_1, \tau) = \sum_{t=1}^{N_f} \sum_{r=1}^{N_r} \left| p_r^{(r)} \right|^2 \left| H_0^{(r,t)} \right|^2.
\]

Since, for typical OSTBCs [2.56], $\bar{p} = p_r^{(m)}$ for $t = 1,2,\ldots,N_f$ and $m = 1,2,\ldots,M$, then
\[
D_{0,m}(\bar{S}_1, \tau) \sim G(N,N_f,N,N_r,\Omega_p) .
\]
This implies $(m_0, \Omega_0) = (N, N_r, \bar{p} \Omega)$. Since $Z_{0,i}$ is formulated as in Lemma 1 with $N_s = 1$, we conclude that $Q(\cdot)$ follows from (2.5.12) with
\[
\phi(s_1(\tau)) = \sum_{m=1}^{M} \left| \eta_0^{(m)} \right|^2 .
\]

### 2.5.5 Spatial Multiplexing MIMO Transmission over Rayleigh Fading - Worst-Case

Let a spatial multiplexing MIMO transmission scheme and a Rayleigh fading channel model. Thus, $N_s = 1$, $M = N_r$, $s_0(1) = \eta_0$, $s_i(1) = \eta_i$ for $i \in \Psi^{(\omega)}$, $\left| \mathbb{E} \left[ H_0^{(r,t)} \right]^2 \right| \sim G(1, \Omega)$ and $\left| \mathbb{E} \left[ H_0^{(r,t)} \right]^2 \right| \sim G(1, \Omega)$ for $t = 1,2,\ldots,N_f$, $r = 1,2,\ldots,N_r$ and $i \in \Psi^{(\omega)}$. Let the interference-oblivious demodulator in (2.5.7) with $\bar{S}_1 = H_0^{(r,m)}$ for $r = 1,2,\ldots,N_r$, where $\bar{m}$ denotes the single stream of the intended link that MT is interested in demodulating. Also, let
\[
\bar{y}(1) = \bar{\psi}(S_1, y(1)) = y(1) , \quad \bar{y}^{(r)}(1) = \bar{\psi}(S_1, s_0(1)) = \sqrt{E / N_f} r_0^{-b} H_0^{(r,m)} \eta_0^{(m)} \quad \text{for} \quad r = 1,2,\ldots,N_r
\]
and $N = N_r$. Thus, unlike the spatial multiplexing MIMO scheme of Section 2.5.5 B, $N_s = 1$ symbols of the intended link are treated as interference. Accordingly, this MIMO scheme is affected by other-cell interference and by IA, i.e., $\sigma_{IA}^2 \neq 0$ in (2.5.3). By inserting (2.5.1) in (2.5.7), we obtain $\Lambda_0^{(m)}(1) = \bar{\eta}_0^{(m)} - \eta_0^{(m)}$, $D_{1,m}(\bar{S}_1, \Delta_0(\tau)) = \left| \bar{\Lambda}_0^{(m)}(1) \right|^2 \sum_{r=1}^{N_r} \left| H_0^{(r,m)} \right|^2$, $D_{2,m}(\bar{S}_1, \Delta_0(\tau), \eta(\tau)) = \left( \Lambda_0^{(m)}(1) \right)^\ast \sum_{r=1}^{N_r} \left( H_0^{(r,m)} \right)^\ast \eta^{(r)}(1)$, $D_{3,m}(\cdot, \cdot, \cdot)$ as shown in (2.5.8) with $Z_{0,i} = \sum_{t=1}^{N_f} \sum_{r=1}^{N_r} H_0^{(r,t)} s_i^{(r)}(1) U_0^{(r)}(1) \text{ and } \bar{U}_0^{(r)}(1) = \left( H_0^{(r,m)} \Lambda_0^{(m)}(1) \right)^\ast$, as well as...
\[ D_{\text{IAI}}(S_1, \Delta_0(\tau)) = |\Delta_0^{(\text{m})}(1)|^2 \sum_{r=1}^{N_r} H_0^{(r,\text{m})} \] and \[ Y_{\text{IAI}} \sim \mathcal{CN}_{\mathcal{P}_0(\tau)}(0, \sigma^2_{\text{IAI}}) \] with \[ \sigma^2_{\text{IAI}}(s_0(\tau)) = \Omega \sum_{\mu=1, \mu \neq n}^{N_s} |s_0^{(\mu)}(1)|^2 = \Omega \sum_{m=1, m \neq n}^{M} |\eta_0^{(m)}|^2 . \] From (2.5.4), we obtain

\[ D_{0,m}(S_1, \Delta_0(\tau)) = |\Delta_0^{(\text{m})}(1)|^2 \sum_{r=1}^{N_r} H_0^{(r,\text{m})} \] and \[ D_{0,m}(S_1) = \sum_{r=1}^{N_r} H_0^{(r,\text{m})} \] . Thus, \[ D_{0,m}(S_1) \sim \mathcal{G}(N_r, N_r, \Omega) \] and \( (m_0, \Omega_0) = (N_r, \Omega) \). Since \( Z_{0,i} \) is formulated as in Lemma 1 with \( N_s = 1 \), we conclude that \( Q(\cdot) \) follows from (2.5.12) with \[ \phi(s_i(1)) = \sum_{i=1}^{N_s} |s_i(1)|^2 . \]

### 2.5.5 Zero-Forcing (ZF) MIMO Receiver over Rayleigh Fading

Let a MIMO transmission scheme with ZF-based reception and a Rayleigh fading channel model [2.57]. Thus, \( N_s \geq N_r \), \( N_s = 1 \), \( M = N_r \), \( s_0(1) = \eta_0 \), \( s_i(1) = \eta_i \) for \( i \in \Psi^{(\text{m})} \),

\[ |H_0^{(r,\text{m})}|^2 \sim \mathcal{G}(1, \Omega) \] and \[ |H_i^{(r,\text{m})}|^2 \sim \mathcal{G}(1, \Omega) \] for \( t = 1, 2, \ldots, N_r \), \( r = 1, 2, \ldots, N_r \) and \( i \in \Psi^{(\text{m})} \). Let the interference-oblivious demodulator in (2.5.3) with \( \tilde{S}_1 = H_0 \)

\[ \tilde{y}(1) = \psi(S_1, y(1)) = \left( H_0^{(\text{m})} H_0 \right)^{-1} H_0^{(\text{m})} y(1) \] and \( N = M = N_r \). Accordingly, the multi-stream demodulator in (2.5.3) is IAI-free with \( \sigma^2_{\text{IAI}} = 0 \) and it can be re-written in the single-stream formulation of (2.5.7), where \( H_0^{(\text{m})} \) denotes the \( (M = N_r) \times N_r \) ZF matrix at the receiver, \( \Delta_0^{(\text{m})}(1) = \tilde{\eta}_0 - \eta_0 \),

\[ D_{1,m}(S_1, \Delta_0(\tau)) = |\Delta_0^{(\text{m})}(1)|^2 \] and \[ D_{2,m}(S_1, \Delta_0(\tau), \mathbf{n}(\tau)) = \left( \Delta_0^{(\text{m})}(1) \right)^2 \sum_{r=1}^{N_r} W_0^{(m,r)} \mathbf{n}(r) \] and \[ D_{3,m}(\cdot, \cdot, \cdot) \] as shown in (2.5.8) with \( Z_{0,i} = \sum_{t=1}^{N_r} \sum_{r=1}^{N_r} H_i^{(r,\text{m})} s_i^{(t)}(1) U_0^{(r)}(1) \) and

\[ U_0^{(r)}(1) = \left( \Delta_0^{(\text{m})}(1) \right)^T W_0^{(m,r)} \] . From (2.5.4), \[ D_{0,m}(S_1, \Delta_0(\tau)) = |\Delta_0^{(\text{m})}(1)|^2 \left( \sum_{t=1}^{N_s} |W_0^{(m,r)}|^2 \right)^{-1} \]

\[ ^{(a)} = \left( \Delta_0^{(\text{m})}(1) \right)^T \left( \tilde{W}_0^{(m,m)} \right)^{-1}, \] where \( \tilde{W}_0 = \left( H_0^{(\text{m})} H_0 \right)^{-1} \) and (a) follows from direct inspection of \( \tilde{W}_0 \).

Thus, \[ D_{0,m}(S_1) = \left( \tilde{W}_0^{(m,m)} \right)^{-1} \sim \mathcal{G}(N_r - N_r + 1, (N_r - N_r + 1), \Omega) \), where (a) follows from [2.57] and [2.58]. This implies \( (m_0, \Omega_0) = (N_r - N_r + 1, \Omega) \). Since \( Z_{0,i} \) is formulated as in Lemma 1 with \( N_s = 1 \), we conclude that \( Q(\cdot) \) follows from (2.5.12) with \[ \phi(s_i(1)) = \sum_{i=1}^{N_s} |s_i(1)|^2 . \]
Let a MIMO transmission scheme with ZF-based precoding and a Rayleigh fading channel model [2.59]. Let \( N_u \) single-antenna MTs be served by intended and interfering BSs in the same channel use within their respective cells. The signal model in (2.5.1) is still applicable with minor changes. With a slight abuse of notation, let \( \mathbf{H}_i \) denote the \( N_u \times N_i \) downlink channel matrix of the links from BS\(_0\) to its \( N_u \) intended MTs. Likewise, let denote by \( \hat{\mathbf{H}}_i \) for \( i \in \Psi^{(u)} \) the \( N_u \times N_i \) downlink channel matrices of the interfering BSs towards the same \( N_u \) MTs as BS\(_0\). Also, let \( \hat{\mathbf{H}}_i \) for \( i \in \Psi^{(u)} \) denote the \( N_i \times N_u \) downlink channel matrix of the links from the \( i \)th interfering BS (BS\(_j\)) towards its intended \( N_u \) single-antenna MTs. In general, \( \hat{\mathbf{H}}_i \neq \hat{\mathbf{H}}_j \). Then, (2.5.1) still holds by replacing \( N_i \) with \( N_u \) and by letting \( N_i \geq N_u \), \( N_r = N_s = 1 \), \( M = N_u \), \( \left| \mathbf{H}_0^{(u,i)} \right|^2 \sim G(1, \Omega) \) and \( \left| \mathbf{H}_i^{(u,i)} \right|^2 \sim G(1, \Omega) \) for \( t = 1, 2, \ldots, N_i \), \( u = 1, 2, \ldots, N_u \) and \( i \in \Psi^{(u)} \). Let \( \mathbf{V}_0 \) denote the \( N_i \times N_u \) precoding matrix used at BS\(_0\), which is defined as \( \mathbf{V}_0 = \mathbf{v}_0 / \| \mathbf{v}_0 \| \) with \( \mathbf{v}_0 = \mathbf{H}_0^{(u)} (\mathbf{H}_0^{(u)} \mathbf{H}_0^{(u)})^{-1} \). Likewise, let \( \hat{\mathbf{V}}_i \) denote the precoding matrix used at BS\(_j\), which is \( \hat{\mathbf{V}}_i = \hat{\mathbf{v}}_i / \| \hat{\mathbf{v}}_i \| \) with \( \hat{\mathbf{v}}_i = \hat{\mathbf{H}}_i^{(u)} (\hat{\mathbf{H}}_i^{(u)} \hat{\mathbf{H}}_i^{(u)})^{-1} \) for \( i \in \Psi^{(u)} \).

Based on these precoding matrices, which assume that the side information available at BS\(_0\) and BS\(_j\) is \( S_0^{(i)} = \mathbf{V}_0 \) and \( S_1^{(i)} = \hat{\mathbf{V}}_i \), respectively, the transmitted vectors are \( \mathbf{s}_0(1) = \Theta(\mathbf{n}_0; S_0^{(i)}) = \mathbf{v}_0 \mathbf{n}_0 \) and \( \mathbf{s}_1(1) = \Theta(\mathbf{n}_1; S_1^{(i)}) = \hat{\mathbf{v}}_i \mathbf{n}_1 \) for \( i \in \Psi^{(u)} \). Let the interference-oblivious demodulator in (2.5.3) with \( S_1 = \| \mathbf{V}_0 \mathbf{n}_0 \| \), \( \mathbf{y}(1) = \hat{\varphi}(\mathbf{s}_1, \mathbf{y}(1)) = \mathbf{y}(1) \), \( \mathbf{y}(1) = \hat{\varphi}(\mathbf{s}_1, \mathbf{s}_0(1)) = \sqrt{E/N} c_0^{-1} \left( \mathbf{n}_0(1)/\| \mathbf{V}_0 \mathbf{n}_0 \| \right) \) and \( N = M = N_u \). Accordingly, the multistream demodulator in (2.5.3) is IA\(1\)-free with \( \sigma_{\text{IAI}}^2 = 0 \) and it can be re-written in the single-stream formulation of (2.5.7) for each intended user \( u = m = 1, 2, \ldots, N_u \). In particular, we have

\[
\Delta_0(1) = \hat{\mathbf{n}}_0 - \mathbf{n}_0
\]

\[
D_{1,m}(S_1, \Delta_0(1)) = \left| \mathbf{A}_0^{(m)}(1) \right|^2 \| \mathbf{V}_0 \mathbf{n}_0 \|^{-2}
\]

\[
D_{2,m}(S_1, \Delta_0(1), \mathbf{n}(1)) = \left( \mathbf{A}_0^{(m)}(1) \right)^* \| \mathbf{V}_0 \mathbf{n}_0 \|^{-1} \mathbf{n}(1)
\]

\[
D_{3,m}(\mathbf{s}(1)) = \sum_{i=1}^{N_i} \mathbf{H}_i^{(m,1)} \mathbf{s}(1) - \mathbf{u}_i^{(m)}
\]

\[
D_{0,m}(S_1, \Delta_0(1)) = \left| \mathbf{A}_0^{(m)}(1) \right|^2 \| \mathbf{V}_0 \mathbf{n}_0 \|^{-2} \quad \text{and} \quad D_{0,m}(S_1) = \| \mathbf{V}_0 \mathbf{n}_0 \|^{-2}
\]

Security: Public
formulated as in Lemma 1 with \( N_s = N_f = 1 \), we conclude that \( Q(\cdot) \) follows from (2.5.12) with 
\[
\phi\left( s_1(1) \right) = \sum_{i=1}^{N_s} \left| s_1^{(i)}(1) \right|^2 = \left\| \hat{V}_0 \eta_0 \right\|^2 \sum_{i=1}^{N_s} \left| s_1^{(i)}(1) \right|^2 = 1, \] 
since \( s_1(1) = \hat{V}_0 \eta_0 \) for \( i \in \Psi^{(0)} \). So far, the analysis for ZF precoding is exact and no approximations have been used. To complete the analysis, however, the distribution of \( D_{0,m}(S_1) = \left\| V_0 \eta_0 \right\|^2 = \left( \eta_0^H V_0^H V_0 \eta_0 \right)^{-1} \) needs to be computed. To the best of our knowledge, however, it is unknown for discrete modulation schemes. To get a tractable yet accurate mathematical framework, we exploit two approximations for the computation of the distribution of \( D_{0,m}(\cdot) \). First of all, we assume that \( \eta_0 \) follows a unit-energy complex Gaussian distribution, i.e., \( \eta_0^{(m)} \sim \mathcal{CN}(0,1) \) for \( m = 1, 2, \ldots, M = N_u \). From [2.54], we obtain 
\[
\eta_0^H V_0^H V_0 \eta_0 \sim N_u \left( N_i - N_u + 1 \right)^{-1} F \left( 2N_u, 2\left( N_i - N_u + 1 \right) \right), \] 
which implies 
\[
D_{0,m}(S_1) \sim \left( \left( N_i - N_u + 1 \right) / N_u \right) F \left( 2\left( N_i - N_u + 1 \right), 2N_u \right). \] 
Second of all, we approximate this resulting scaled F-distribution with a scaled Chi-Square distribution, i.e., 
\[
D_{0,m}(S_1) \sim (2N_u)^{-1} \chi^2_{2\left( N_i - N_u + 1 \right)}, \] 
which is known to be accurate for \( N_u \geq 1 \) and, in turn, can be re-written in terms of a Gamma distribution, i.e., 
\[
D_{0,m}(S_1) \sim G\left( N_i - N_u + 1, N_u^{-1} \left( N_i - N_u + 1 \right) \Omega \right). \] 
This implies \( (m_0, \Omega_0) = \left( N_i - N_u + 1, \Omega / N_u \right) \).

In Section 2.5.6, these approximations are shown to be accurate enough for typical MIMO setups.

### 2.5.5 Numerical and Simulation Results

In this section, numerical examples are shown to substantiate the accuracy of the mathematical frameworks and to confirm the performance trends. The frameworks are compared against Monte Carlo simulations, which are obtained by using the procedure described in [2.10], [2.55]. The accuracy of the PPP-based abstraction for modeling the error performance of cellular networks is investigated by comparing it with grid-based abstraction models. Hence, similar curves are not reported in this section. The simulation setup is summarized in the caption of each figure, where markers show Monte Carlo simulations, solid lines the analytical framework and dashed lines the asymptotic framework. As for the implementation of the mathematical frameworks for QAM with \( M \geq 4 \), the parameters as follows are used: \( \alpha_1 = \pi / 2, \beta_1 = 3 \left( M - 1 \right), \gamma_1 = \pi / 2, \delta_1 = 4 \left( \sqrt{M} - 1 \right) / \sqrt{M} \) and \( \alpha_2 = \pi / 4 \).
\[ \beta_2 = \frac{6}{(M-1)}, \quad \gamma_2 = \frac{\pi}{4}, \quad \delta_2 = 4\left(\sqrt{M-1}\right)^2 / M. \]

If \( M = 2 \), the quadruplet \( (\alpha, \beta, \gamma, \delta) = \left(\frac{\pi}{2}, 2, \frac{\pi}{2}, 1\right) \) is used.

Selected numerical examples are illustrated in Figs 2.5.1-2.5.6, where the ASEP is depicted as a function of \( E/N_0 \), which is a reference signal-to-noise-ratio that is computed at a fixed reference distance of one meter from the transmitter. These figures confirm the accuracy of the proposed mathematical frameworks. The approximations proposed in Section 2.5.5 are confirmed to be sufficiently accurate in the considered setup. Similar accuracies are obtained for different parameters.

This confirms that the impact of the fading severity is negligible in the presence of other-cell interference. The path-loss exponent has a different impact in noise- and interference-limited regimes. In particular, a bigger path-loss is beneficial in interference-limited cellular networks, since the other-cell interference is reduced. Figure 2.5.2 (b) shows a similar behavior in the presence of receive-diversity. Figure 2.5.2 (a) confirms that receive-diversity is still beneficial, but the gain in the presence of other-cell interference is reduced compared to the noise-limited scenario.
Figure 2.5.3 shows that the performance gain offered by transmit-diversity compared to receive-diversity in noise-limited networks is not observable in the presence of other-cell interference. In fact, the ASEP of Fig. 2.5.2(a) and Fig. 2.5.3(a) is almost the same.

Figure 2.5.4 shows the ASEP of spatial multiplexing MIMO and it brings to our attention the detrimental impact of IAI, even in the presence of strong other-cell interference. In general, we observe that increasing the number of antennas is beneficial if a multi-stream
demodulator is used (see Fig. 2.5.4 (a)). By comparing Fig. 2.5.4 (a) with Fig. 2.5.3 (a) for $N_r = 2$, which provide the same rate, we observe that spatial multiplexing provides better performance than the Alamouti code, but at the cost of a higher demodulation complexity.

![Fig. 2.5.4: ASEP as a function of Nt, Nr and modulation order](image)

Figure 2.5.5 compares ZF reception and ZF precoding under similar operating conditions, and under the assumption that $M$ is independent of $N_t$ and $N_u$, respectively. The figure confirms that the ASEP gets worse by increasing $N_t$ and $N_u$. A close inspection of Figs. 2.5.5(a) and 2.5.5(b) reveals that ZF reception and precoding provide almost the same performance in the interference-limited regime. Architectural design and mod/demodulation complexity are, however, quite different between them.

Figure 2.5.6 provides a sound confirmation of some non-trivial trends highlighted in [2.55]. Figure 2.5.6(a) shows that the ASEP may get worse by increasing $N_t$, while Fig. 2.5.6(b) shows that the ASEP gets better by increasing $N_t$. The trend in Fig. 2.5.6 (a) originates from the fact that $K_{\text{PSK}}^{(1)}$ decreases by increasing $N_t$ and that this effect is not counterbalanced by the reduction of the modulation order $M$. On the other hand, the trend in Fig. 2.5.6 (b) follows because $K_{\text{PSK}}^{(1)}$ is kept fixed by increasing $N_t$. As a result, reducing the modulation order $M$ is beneficial ($K_{\text{PSK}}^{(2)}$ decreases). By comparing Fig. 2.5.6 (a) with Fig. 2.5.4 (a) for
$N_r = 4$ (same MIMO setup and rate), the performance vs. complexity trade-off between spatial multiplexing with ML-optimum demodulation and ZF-based reception clearly emerges in the interference-limited regime.

In conclusion, the proposed mathematical frameworks are sufficiently accurate and insightful to the analysis, design and optimization of MIMO-aided cellular networks.

---

**Fig. 2.5.5**: ASEP as a function of $N_t$ and $N_u$

**Fig. 2.5.6**: ASEP as a function of $N_t$, $N_r$ and modulation order
3. Performance Analysis of Uplink Heterogeneous Cellular Networks

3.1 Introduction
The use of PPP-based abstraction model for modeling heterogeneous networks and derivation of the corresponding downlink coverage and rate under various association and interference coordination strategies has been extensively explored, several notable examples can be found in Section 2 and reference therein.

However, the analysis of the uplink by PPP-based abstraction model is limited, because the interfering MTs in uplink are not strictly Poisson distributed nodes and the transmit power of an interfering MT is correlated with its path loss to the BS due to the uplink power control. Several examples on uplink cellular analysis by utilizing stochastic geometry can be found in [3.1][3.2][3.3]. However, there is no studies on performance in the uplink with multi-antenna receiver to the best of authors knowledge.

In this section we provide a mathematical framework to study the coverage and rate performance cellular uplinks with fractional power control and then extend the analysis to multi-antenna receiver and heterogeneous networks.

3.2 Stochastic geometry modeling and analysis of the coverage probability and average rate of single-tier uplink cellular networks

3.2.1 System Model and Problem Formulation
In this section, both the MTs and BSs are equipped with single transmit/receive antenna. We assume that the BSs are modeled as points of a homogeneous PPP ($\Phi_1$) of density $\lambda_1$ and that the MTs are modeled as points of a homogeneous PPP of density $\lambda_{\text{MT}}$. Each MT is assumed to be connected to its nearest BS. The probe BS is denoted by $\text{BS}_0$. Some important assumptions are made to study the uplink performance according to [3.1]: i)Full load assumption, which means that $\lambda_{\text{MT}} \gg \lambda_1$ and all BSs are active; ii) Each BS selects only one MT to serve at one resource block; iii) Correlations between MTs are neglected and active MTs approximately form a PPP with same density as BS density.
The PDF \( f_{R_i} (\xi) \) and CCDF \( \bar{F}_{R_i} (\xi) \) of the closest distance between a MT and BSs are as follows:

\[
f_{R_i} (\xi) = 2\pi\lambda_i \xi \exp\left(-\pi\lambda_i \xi^2 \right) \\
\bar{F}_{R_i} (\xi) = \exp\left(-\pi\lambda_i \xi^2 \right)
\]

(3.2.1)

The MTs apply truncated fractional power control [3.1][3.2][3.3], mathematically the transmit power of MTs can be formulated as follows:

\[
P_{TX,i} = \begin{cases} 
p_i \left( \tilde{R}_i \right)^{\alpha_1 \varepsilon_1}, & \text{if } p_i \left( \tilde{R}_i \right)^{\alpha_1 \varepsilon_1} < P_{MT} \\
0, & \text{otherwise} \end{cases}
\]

(3.2.2)

where \( p_i \) is the pre-defined reference power, \( \alpha_1 \) is the path-loss exponent, \( \varepsilon_1 \) is the fractional factor, \( P_{MT} \) is the maximum allowed transmit power, and \( \tilde{R}_i \) denotes the closest distance conditioned on connection, whose distribution is as follows:

\[
f_{R_i} (\xi) = \frac{f_{R_i} (\xi)}{\Pr(X_i)} , \xi \in \left[ 0, \left( \frac{P_{MT}}{p_i} \right)^{\frac{1}{\alpha_1 \varepsilon_1}} \right]
\]

(3.2.3)

Let the event \( X_i \) denote connection, the probability of connection \( \Pr(X_i) \) is:

\[
\Pr(X_i) = \int_{0}^{\left( \frac{P_{MT}}{p_i} \right)^{\frac{1}{\alpha_1 \varepsilon_1}}} f_{R_i} (\xi) d\xi = 1 - \exp \left(-\pi\lambda_i \left( \frac{P_{MT}}{p_i} \right)^{\frac{2}{\alpha_1 \varepsilon_1}} \right)
\]

(3.2.4)

With fractional power control, the received signal at the probe BS in this uplink cellular network is as follows:

\[
y_{1,0} = \sqrt{p_i \tilde{R}_i^{\alpha_1 (\varepsilon_1 - 1)}} s_{1,0} + \sum_{i \in \Phi_1^0} \sqrt{p_i \tilde{R}_i^{\alpha_1 \varepsilon_1} D_{1,i}^{\alpha_0} h_{1,i} s_{1,i}} 1(D_{1,i} > \tilde{R}_i) + n_{1,0}
\]

(3.2.5)

where \( D_{1,i} \) denotes the distance to probe BS from \( i \) th interfering MTs, \( \Phi_1^0 \) denotes the set of interfering MTs, which is still approximately PPP with density \( \lambda_i \). The indicator function \( 1(D_{1,i} > \tilde{R}_i) \) is the constraint due to shortest-distance cell association in the uplink. The noise \( n_{1,0} \) is AWGN with noise power \( \sigma^2 \).

The SINR of this uplink cellular network can be formulated as follows:

\[
\text{SINR} = \frac{U_{1,0}}{\sigma^2 + I_{1,i}^0}
\]

(3.2.6)
where:

$$U_{1,0} = p_i R_{1,0}^{\alpha_i} |h_{i,0}|^2$$

$$I_{1,1} = \sum_{i \in \Phi_1^0} p_i R_{1,1}^{\alpha_i} D_{i}^{\alpha_i} |h_{i,1}|^2 1(D_{i} > R_{1,1})$$  \hspace{1cm} (3.2.7)

where $U_{1,0}$ is useful signal power and $I_{1,1}$ is the aggregate other-cell interference, $|h_{i,0}|^2$ and $|h_{i,1}|^2$ for $i \in \Phi_1^0$ are the per-link power gains of intended and interfering links, which are assumed to be Gamma distributed $\text{Gamma}(1,1)$, i.e., Rayleigh fading channel model is used.

The coverage probability ($P_{\text{cov}}$) and average rate ($R$) are studied. They can be formulated as follows:

$$P_{\text{cov}}(\gamma) = \Pr\{\text{SINR} \geq \gamma\}$$ \hspace{1cm} (3.2.8)

$$R = E\{\ln(1 + \text{SINR})\} = \int_0^\infty P_{\text{cov}}(t-1) dt = -\int_0^\infty \ln(1 + y) P_{\text{cov}}(y) dy$$ \hspace{1cm} (3.2.9)

where $\gamma > 0$ is a reliability threshold, (a) follows from [2.1] and (b) follows by applying integration by parts, since $P_{\text{cov}}(\gamma \to 0) = 1$ and $P_{\text{cov}}(\gamma \to \infty) = 0$.

### 3.2.2 Gil-Pelaez Based Mathematical Modeling

Mathematical frameworks to the computation of (3.2.8) and (3.2.9) are provided, by assuming that $|h_{i,0}|^2$ and $|h_{i,1}|^2$ for $i \in \Phi_1^0$ as independent but identically distributed Gamma RVs.

#### Theorem 3.2.1

If single tier SISO cellular uplink, the SINR coverage probability is as follows:

$$P_{\text{cov}}(\gamma) = \frac{1}{2} \Pr(X_1) - \frac{1}{\pi} \int_0^\infty \text{Im} \left[ Q_i \left( j \frac{P_i}{\gamma} \omega \right) e^{i \omega \sigma^2} CF_{i_1}^{\omega} (\omega) \right] \frac{d\omega}{\omega}$$ \hspace{1cm} (3.2.10)

where the functions are computed as:

$$Q_i(x) = \int_0^{(\frac{2}{\alpha_i})^{\frac{1}{\alpha_i}}} \left( 1 + x \bar{\varepsilon}^{\alpha_i-1} \right)^{-1} 2\pi \lambda_i \bar{\varepsilon} \exp\left(-\pi \lambda_i \bar{\varepsilon}^2\right) d\bar{\varepsilon}$$

$$CF_{i_1}^{\omega} (\omega) = \exp \left[ \lambda_i \pi \int_0^{(\frac{2}{\alpha_i})^{\frac{1}{\alpha_i}}} \left[ 1 - F_1 \left( -\frac{2}{\alpha_i} \frac{1}{1-\frac{2}{\alpha_i}}, 1-\frac{2}{\alpha_i}, j \omega p_i x^{\alpha_i} \right) \right] \frac{2\pi \lambda_i x^3 \exp\left(-\pi \lambda_i x^2\right) \Pr(X_1)}{dx} \right]$$ \hspace{1cm} (3.2.11)

**Proof:** Theorem 3.2.1 can be derived by applying Gil-Peleaz inversion as in Section 2.3. □
**Theorem 3.2.2**

If single tier SISO cellular uplink, the average rate can be computed as:

\[
R = \frac{1}{\pi} \int_0^\infty \text{Im} \left[ e^{i\omega^2} CF_{\xi_1} (\omega) \bar{f}_R (\xi) Z_1 (\omega, \xi) \right] \frac{d\omega}{\omega} d\xi \tag{3.2.12}
\]

where:

\[
Z_1 (\omega, \xi) = \int_0^\infty \ln (y + 1) \frac{jp_i \omega \xi^{\alpha(\xi^{-1})}}{y^2} \left(1 + j \frac{p_i \omega \xi^{\alpha(\xi^{-1})}}{y} \right)^{-2} dy
\]

\[
jp_i \omega \xi^{\alpha(\xi^{-1})} \ln \left(jp_i \omega \xi^{\alpha(\xi^{-1})}\right)
= \frac{jp_i \omega \xi^{\alpha(\xi^{-1})} \ln \left(jp_i \omega \xi^{\alpha(\xi^{-1})}\right)}{jp_i \omega \xi^{\alpha(\xi^{-1})} - 1}
\tag{3.2.13}
\]

\[
\bar{f}_R (\xi) = \frac{f_R (\xi)}{\text{Pr}(X_i)}
\]

**Proof:** Applying integral by parts we have

\[
R = \mathbb{E} \left\{ \ln (1 + \text{SINR}) \right\} = \int_0^\infty p_{\text{cov}} (y) \frac{1}{y + 1} dy = -\int_0^\infty \ln (y + 1) \left(p_{\text{cov}} (y)\right) dy
\]

\[
\left(p_{\text{cov}} (y)\right) = -\frac{1}{\pi} \int_0^\infty \text{Im} \left[ e^{i\omega^2} CF_{\xi_1} (\omega) \tilde{Q}_i (\omega, y) \right] \frac{d\omega}{\omega}
\tag{3.2.14}
\]

\[
\tilde{Q}_i (\omega, y) = \frac{jp_i \omega \xi^{\alpha(\xi^{-1})} N_R}{y^2} \int_0^\infty \left(1 + j \frac{p_i \omega \xi^{\alpha(\xi^{-1})}}{y} \right)^{-2} \bar{f}_R (\xi) d\xi
\]

And the rest closed-form is with the aid of the Mellin-Barnes theorem in[3.8]. □

### 3.3 Extension to multi-antenna uplink cellular networks with MRC receiver

#### 3.3.1 System Model

The same system models and assumptions as in Section 3.2.1 hold. Similar to (3.2.5), but assuming that all BSs are all equipped with \(N_r\) receive antennas, the receive signal at the probe BS is as:

\[
y_{1,0} = \sqrt{p_i \tilde{R}_{1,0}^{\alpha(\xi^{-1})}} h_{1,0}s_{1,0} + \sum_{i \in \Phi_i^0} \sqrt{p_i \tilde{R}_{1,i}^{\alpha(\xi^{-1})}} D_{1,i} h_{1,i}s_{1,i} 1(D_{1,i} > \tilde{R}_{1,i}) + n_{1,0}
\tag{3.3.1}
\]

where \(h_{1,0}\) and \(h_{1,i}\) are channel vectors whose entries are assumed to be i.i.d. complex Gaussian RVs, i.e., Rayleigh fading channel models are applied. \(n_{1,0}\) is AWGN noise vector with per dimension noise power \(\sigma^2\).

If MRC receiver applied, the weight vector is \(h_{1,0}^* [3.4]\) and the post-processing signal is as:
\[ z_{1,0} = \mathbf{h}_{1,0}^* \mathbf{y}_{1,0} \]
\[ = \sqrt{p_i R_{1,0}^{\rho_{e_i}}(\mathcal{e}_{i}^{-1})} \mathbf{h}_{1,0}^* \mathbf{h}_{1,0} \mathbf{s}_{1,0} + \sum_{i \in \Phi_i^0} \sqrt{p_i R_{1,j}^{\rho_{e_i}}(\mathcal{e}_{i}^{-1})} \mathbf{h}_{1,0}^* \mathbf{h}_{1,0} s_{1,j} \mathcal{1} \{D_{1,j} > \tilde{R}_{1,j}\} + \mathbf{h}_{1,0}^* \mathbf{n}_{1,0} \] (3.3.2)

The SINR can be formulated as follows [3.4]:
\[ \text{SINR} = \frac{U_{1,0}}{\sigma^2 + I_{1,1}^0} \] (3.3.3)

where:
\[ U_{1,0} = p_i R_{1,0}^{\rho_{e_i}}(\mathcal{e}_{i}^{-1}) \left\| \mathbf{h}_{1,0} \right\|^2 \]
\[ I_{1,1}^0 = \sum_{i \in \Phi_i^0} p_i R_{1,j}^{\rho_{e_i}}(\mathcal{e}_{i}^{-1}) \left\| \mathbf{h}_{1,0}^* \mathbf{h}_{1,0} \right\| \mathcal{1} \{D_{1,j} > \tilde{R}_{1,j}\} \] (3.3.4)

\[ \left\| \mathbf{h}_{1,0} \right\|^2 \text{ and } \left\| \mathbf{h}_{1,0}^* \mathbf{h}_{1,j} \right\|^2 \]
are independent RVs:
\[ \left\| \mathbf{h}_{1,0} \right\|^2 \sim \text{Gamma} \left( N_r, N_r \right) \]
\[ \left\| \mathbf{h}_{1,0}^* \mathbf{h}_{1,j} \right\|^2 \sim \text{Gamma}(1,1) \] (3.3.5)

**Proof:**
\[ \left\| \mathbf{h}_{1,0} \right\|^2 = \sum_{r=1}^{N_r} |h_{1,0}^{(r)}|^2 \sim \sum_{r=1}^{N_r} \text{Gamma}(1,1) \sim \text{Gamma} \left( N_r, N_r \right) \]

Conditioned on \( \mathbf{h}_{1,0} \), \( \left\| \mathbf{h}_{1,0}^* \mathbf{h}_{1,j} \right\|^2 \) is a linear combination of complex Gaussian random variables because \( \mathbf{h}_{1,0} \) and \( \mathbf{h}_{1,j} \) are independent, so that
\[ \left\| \mathbf{h}_{1,0}^* \mathbf{h}_{1,j} \right\|^2 \]
conditioned on \( \mathbf{h}_{1,0} \) is complex Gaussian. Compute the mean and variance of
\[ \left\| \mathbf{h}_{1,0}^* \mathbf{h}_{1,j} \right\|^2 \]
conditioned on \( \mathbf{h}_{1,0} \):
\[ E \left\{ \left\| \mathbf{h}_{1,0}^* \mathbf{h}_{1,j} \right\|^2 \right\} = \mathbf{h}_{1,0}^* E \left\{ \left\| \mathbf{h}_{1,0} \right\|^2 \right\} \mathbf{h}_{1,0} = \mathbf{h}_{1,0}^* E \left\{ \mathbf{h}_{1,j} \right\} = 0 \] and
\[ E \left\{ \left\| \mathbf{h}_{1,0}^* \mathbf{h}_{1,j} \right\|^2 \right\} = \mathbf{h}_{1,0}^* E \left\{ \left\| \mathbf{h}_{1,0} \right\|^2 \right\} \mathbf{h}_{1,j} = 1 \] respectively, so that the PDF of
\[ \left\| \mathbf{h}_{1,0}^* \mathbf{h}_{1,j} \right\|^2 \]
conditioned on \( \mathbf{h}_{1,0} \) can be shown as
\[ f_{\left\| \mathbf{h}_{1,0}^* \mathbf{h}_{1,j} \right\|^2 | \mathbf{h}_{1,0}} (x | \mathbf{h}_{1,0}) = \frac{1}{\pi} \exp \left( - \frac{\left\| \mathbf{h}_{1,0}^* \mathbf{h}_{1,j} \right\|^2}{\left\| \mathbf{h}_{1,0} \right\|^2} \right) \], which is
independent of $\mathbf{h}_{i,0}$. So that $\left| \mathbf{h}_{i,0}^* \right| \mathbf{h}_{i,j} \sim \text{Gamma}(1,1)$, which is independent with $\mathbf{h}_{i,0}$. □

### 3.3.2 Gil-Pelaez Based Mathematical Modeling

Similar to Theorem 3.2.1 and 3.2.2, mathematical frameworks to the computation of (3.2.8) and (3.2.9) are provided, by having $\left| \mathbf{h}_{i,0} \right|^2$ and $\left| \mathbf{h}_{i,0}^* \mathbf{h}_{i,j} \right|^2$ for $i \in \Phi_1^0$ as independent but identically distributed Gamma RVs.

#### Theorem 3.3.1

In single tier uplink cellular networks, if a MRC receiver is used, the SINR coverage probability is as follows:

$$P_{\text{cov}}(\gamma) = \frac{1}{2} \Pr(X_1) - \frac{1}{\pi} \int_0^\infty \text{Im} \left[ \frac{1}{\sqrt{\pi}} \frac{1}{\sqrt{\pi}} \exp(-\pi \lambda_1 \xi^2) \right] d\xi$$

where the functions are computed as:

$$\bar{Q}_1(x) = \frac{1}{\sqrt{\pi}} \frac{1}{\sqrt{\pi}} \exp(-x^2)$$

$$CF_{I_1}(\omega) = \exp \left( \lambda_1 \pi \int_0^{\pi/2} \left[ 1 - 2F1 \left( -\frac{2}{\alpha_i}, 1, -\frac{2}{\alpha_i} \right) \frac{\omega p_j x^\alpha_{(k-1)}}{\Pr(X_1)} \right] d\omega \right)$$

**Proof:** Theorem 3.3.1 can be derived by applying Gil-Peleaz inversion as in Section 2.3. □

#### Theorem 3.3.2

In single tier uplink cellular networks, if a MRC receiver is used, the average rate can be computed as:

$$R = \frac{1}{\pi} \int_{\xi_0}^\infty \int_0^\infty \text{Im} \left[ e^{i\omega r} CF_{I_1}(\omega) \bar{f}_{\mathbf{h}_i}(\xi) Z_1(\omega, \xi) \right] d\omega d\xi$$

where:

$$Z_1(\omega, \xi) = \int_0^\infty \ln \left( y + 1 + j p_l \omega \xi^\alpha_{(k-1)} N_l \right) y^{N_i - 1} d\eta$$

$$\bar{f}_{\mathbf{h}_i}(\xi) = \frac{\int_{\mathbf{h}_i}(\xi)}{\Pr(X_1)}$$

With integer-valued $N_i$, $Z_1(\omega, \xi)$ a has closed-form exists, as follows:
\[
N_r = 2, Z_1(\omega, \xi) = \frac{A(-1 + A + (-2 + A)\ln(A))}{(-1 + A)^2}
\]

\[
N_r = 3, Z_1(\omega, \xi) = \frac{A(5 - 8A + 3A^2 + 2(3 + (-3 + A)A)\ln(A))}{2(-1 + A)^3}
\]

\[
N_r = 4, Z_1(\omega, \xi) = \frac{A((-1 + A)(26 + A(-31 + 11A)) + 6(-2 + A)(2 + (-2 + A)A)\ln(A))}{6(-1 + A)^4}
\]

(3.3.10)

where we use short-hands \( A = j_p \omega \mu_\xi^{(\xi_j - 1)} \) for simplicity.

**Proof:** See proof of Theorem 3.2.2. □

### 3.4 Extension to multi-tier heterogeneous uplink cellular networks

#### 3.4.1 System Model and Problem Formulation

Assume a \( K \) tiers uplink cellular network, where the MTs are equipped with single transmit antenna and that BSs are equipped with \( N_r \) receive antennas and apply MRC receiver. BSs locations in each tier can be modeled by PPPs with densities \( \lambda_j, j = 1, \ldots, K \). MTs are modeled by a PPP with density \( \lambda_{MT} \). Same assumptions as in Section 3.2.1 and 3.3.1 hold.

PDF and CCDF of the closest distance between MT and tier \( j \) BSs are as follows:

\[
f_{R_j}(\xi) = 2\pi \lambda_j \xi \exp(-\pi \lambda_j \xi^2)
\]

\[
F_{R_j}(\xi) = \exp(-\pi \lambda_j \xi^2)
\]

(3.4.1)

**Cell Association**

The association being in tier \( j \) is based on [3.5]:

\[
j : \arg \max_{k=1, \ldots, K} \left( T_k R_{k,0}^{-\alpha_k} \right)
\]

(3.4.2)

Where \( \alpha_k \) is the path-loss exponent per tier, \( R_{k,0} \) is the closest distance between MT and 0th BS in tier \( k \) and PDF and CCDF of \( R_{k,0} \) are the same as defined in . \( T_k \) denotes the association weight and three different association schemes can be studied and compared.

**Association 1:** based on the received power in the downlink.

\[
T_k = P_k
\]

where \( P_k \) denotes tier \( k \) BS transmit power.
Association 2: based on best biased received power in the downlink.

\[ T_k = P_k B_k \]

where \( B_k \) denotes tier \( k \) bias factor.

Association 3: smallest path loss association.

\[ T_k = 1 \]

Let \( X_j \) denote the event that cell association being with tier \( j \). The probability \( \Pr(X_j) \) denotes the association probability, which can be computed as follows:

\[
\Pr(X_j) = \Pr(T_k R_{j,k}^{\alpha_j} > T_k R_{k,j}^{\alpha_k}, \forall k \neq j)
\]  

(3.4.3)

The CDF and PDF of \( R_j \), which is the shortest distance conditioned on tier \( j \) association, can be formulated as follows.

\[
F_{R_j}(\xi) = \int_0^\xi f_{R_j}(R_j) \prod_{k=1 \wedge j}^k \frac{F_{R_k}\left(T_k R_{j,k}^{\alpha_j}\right)^{1/\alpha_k}}{f_{R_k}\left(R_k\right)} dR_j, \xi \in \left[0, \left(\frac{P_{MT}}{P_j}\right)^{1/\alpha_{j,k}}\right]
\]

(3.4.4)

\[
f_{R_j}(R_j) = \frac{f_{R_j}(R_j) \prod_{k=1 \wedge j}^k \frac{F_{R_k}\left(T_k R_{j,k}^{\alpha_j}\right)^{1/\alpha_k}}{f_{R_k}\left(R_k\right)}}{\int_0^\xi f_{R_j}(R_j) \prod_{k=1 \wedge j}^k \frac{F_{R_k}\left(T_k R_{j,k}^{\alpha_j}\right)^{1/\alpha_k}}{f_{R_k}\left(R_k\right)} dR_j}, R_j \in \left[0, \left(\frac{P_{MT}}{P_j}\right)^{1/\alpha_{j,k}}\right]
\]

Proof: it follows the definition of conditional probability:

\[
F_{R_j}(\xi) = \frac{\Pr(R_j < \xi, X_j)}{\Pr\left(R_j < \left(\frac{P_{MT}}{P_j}\right)^{1/\alpha_{j,k}}, X_j\right)}, \xi \in \left[0, \left(\frac{P_{MT}}{P_j}\right)^{1/\alpha_{j,k}}\right]
\]

(3.4.5)

\[
= \frac{\Pr\left(R_j < \xi, R_k > \left(\frac{T_k R_{j,k}^{\alpha_k}}{T_j}\right)^{1/\alpha_k}, \forall k \neq j\right)}{\Pr\left(R_j < \left(\frac{P_{MT}}{P_j}\right)^{1/\alpha_{j,k}}, R_k > \left(\frac{T_k R_{j,k}^{\alpha_k}}{T_j}\right)^{1/\alpha_k}, \forall k \neq j\right)}, \xi \in \left[0, \left(\frac{P_{MT}}{P_j}\right)^{1/\alpha_{j,k}}\right]
\]

This conclude the proof \( \square \)
As in (3.2.2), fractional power control scheme defines the MT transmit power conditioned on a MT being associated with tier j of BS:

\[
P_{TX, j} = \begin{cases} 
    p_j \left( \frac{R_j}{R_{j,0}} \right)^{\alpha_j \varepsilon_j}, & \text{if } p_j \left( \frac{R_j}{R_{j,0}} \right)^{\alpha_j \varepsilon_j} < P_{MT} \\
    0, & \text{otherwise}
\end{cases}
\]  

(3.4.6)

where \( p_j \) is the pre-defined reference power per tier, \( \alpha_j \) is the path-loss exponent per tier, \( \varepsilon_j \) is the fractional factor per tier.

**Signal and System Model**

Similar to (3.3.1), but assuming K tier cellular uplink, the receive signal at the probe BS is as:

\[
y_{j,0} = \sqrt{p_j R_{j,0}^{\alpha_j \varepsilon_j}} h_{j,0}^* s_{j,0} + n_{j,0} \\
+ \sum_{i \in \Phi_j^0} p_j R_{j,i}^{\alpha_j \varepsilon_j} D_{j,i}^{-\alpha_j} h_{j,i}^* s_{j,i} 1\left(D_{j,i} > \tilde{R}_{j,i}\right) \\
+ \sum_{k=1}^{K} \sum_{i \in \Phi_k} \sqrt{p_k R_{k,j}^{\alpha_k \varepsilon_k}} D_{k,j}^{-\alpha_k} h_{k,j}^* s_{k,j} 1\left(D_{k,j} > \frac{T_j}{T_k} \tilde{R}_{k,j}^{\alpha_k \varepsilon_k}\right) 
\]

(3.4.7)

where \( \Phi_k \) denotes the set of active MTs in tier k, which is PPP with density \( \lambda_k \). \( D \) denotes the distance to probe BS from interfering MT. The indicator functions \( 1\left(D_{j,i} > \tilde{R}_{j,i}\right) \) and \( 1\left(D_{k,j} > \frac{T_j}{T_k} \tilde{R}_{k,j}^{\alpha_k \varepsilon_k}\right) \) are the constraints due to cell association in the uplink. Rayleigh fading model is assumed, i.e., \( h \sim \mathcal{CN} (0, I_{N_k}) \). The noise vector is AWGN with per dimension noise power \( \sigma^2 \).

The MRC receiver multiplies a weight vector \( h_{j,0}^* \in \mathbb{C}^{1 \times N} \), for the received signal \( y_{j,0} \). Thus, the post-processed signal is:

\[
z_{j,0} = h_{j,0}^* y_{j,0} \\
= \sqrt{p_j R_{j,0}^{\alpha_j \varepsilon_j}} h_{j,0}^* h_{j,0}^* s_{j,0} + h_{j,0}^* n_{j,0} \\
+ \sum_{i \in \Phi_j^0} p_j R_{j,i}^{\alpha_j \varepsilon_j} D_{j,i}^{-\alpha_j} h_{j,i}^* h_{j,0}^* s_{j,i} 1\left(D_{j,i} > \tilde{R}_{j,i}\right) \\
+ \sum_{k=1}^{K} \sum_{i \in \Phi_k} \sqrt{p_k R_{k,j}^{\alpha_k \varepsilon_k}} D_{k,j}^{-\alpha_k} h_{k,j}^* h_{k,0}^* s_{k,j} 1\left(D_{k,j} > \frac{T_j}{T_k} \tilde{R}_{k,j}^{\alpha_k \varepsilon_k}\right) 
\]

(3.4.8)

The output SINR can then be formulated as:
\[ \text{SINR}_j = \frac{U_{j,0}}{\sigma^2 + I_{j,0}^0 + \sum_{k=1}^{K} I_{k,j}} \]  

(3.4.9)

where the short-hands follow as used:

\[ U_{j,0} = p_j \bar{R}_{j,0} \left\| h_{j,0} \right\|^2 \]

\[ I_{j,j} = \sum_{i \in \Phi_j^0} p_j \bar{R}_{j,i} D_{j,i} \left\| h_{j,0} \right\|^2 \left( D_{j,i} > \bar{R}_{j,i} \right) \]

\[ I_{k,j} = \sum_{i \in \Phi_k} p_k \bar{R}_{k,i} D_{k,i} \left\| h_{j,0} \right\|^2 \left( D_{j,i} > \bar{T}_{k,i} \bar{R}_{k,i} \right) \]

\[ \left\| h_{j,0} \right\|^2 \sim \text{Gamma}(N_e, N_e) , \quad \left\| h_{j,i} \right\|^2 \sim \text{Gamma}(1,1) \quad \text{and} \quad \left\| g_{k,i} \right\|^2 \sim \text{Gamma}(1,1) \]

are independent Gamma RVs.

\[ \text{Proof:} \text{ See proof of (3.3.5).} \]

**Problem Statement**

We focus on the coverage probability:

\[ P_{\text{cov}}(\gamma) = \sum_{j=1}^{K} P_{\text{cov}}(\gamma|X_j) \Pr(X_j) \]  

(3.4.11)

where the per tier coverage is defined as:

\[ P_{\text{cov}}(\gamma|X_j) = \Pr(\text{SINR}_j > \gamma) \]  

(3.4.12)

**3.4.2 Gil-Pelaez Based Mathematical Modeling**

Similar to Theorem 3.2.1 and 3.3.1, mathematical frameworks to the computation of (3.4.11) are provided, by having \( \left\| h_{j,0} \right\|^2 \), \( \left\| h_{j,i} \right\|^2 \) for \( i \in \Phi_j^0 \) and \( \left\| g_{k,i} \right\|^2 \) for \( i \in \Phi_k \) as independent but identically distributed Gamma RVs.

**Theorem 3.4.1**

Let SINR in (3.4.10), the general framework to compute SINR coverage probability in the uplink SIMO networks with MRC receiver can be summarized as follows:

\[ P_{\text{cov}}(\gamma) = \frac{1}{2} \sum_{j=1}^{K} \Pr(X_j) - \frac{1}{2} \sum_{j=1}^{K} \int_0^{\infty} \text{Im} \left[ \bar{Q}_j \left( j \frac{P_j}{T} \omega \right) e^{j \omega \sigma^2} CF_{l_{j,0}}(\omega) \prod_{k=1}^{K} CF_{l_{k,j}}(\omega) \right] d\omega \]  

(3.4.13)
where:

\[
\tilde{Q}_j(x) = \int \left(1 + x \xi \right)^{-N_j} \tilde{f}_{R_j}(\xi) d\xi
\]

\[
CF_{j\mu_j}(\omega) = \exp \left( \lambda_j \pi \int x^2 \left[1 - 2 F_1 \left(-\frac{2}{\alpha_j},1,1 - \frac{2}{\alpha_j}, j\omega p_j x^{\alpha_j(\epsilon_j{-}1)} \right) \right] f_{R_j}(x) dx \right)
\]

(3.4.14)

\[
CF_{k\mu_j}(\omega) = \exp \left( \lambda_k \pi \int \left(\frac{T_j}{T_k} x^{\alpha_j} \right)^{2/\alpha_j} \left[1 - 2 F_1 \left(-\frac{2}{\alpha_j},1,1 - \frac{2}{\alpha_j}, j\omega p_j x^{\alpha_j(\epsilon_j{-}1)} \right) \right] f_{R_j}(x) dx \right)
\]

And the association probability:

\[
\Pr(X_j) = \int f_{R_j}(x) \prod_{k=1}^k \tilde{F}_{R_k} \left( \frac{T_k}{T_j} x^{\alpha_j} \right)^{1/\alpha_j} dx
\]

(3.4.15)

with \( \tilde{f}_{R_j}(\xi) = \frac{f_{R_j}(\xi)}{\Pr(X_j)} \)

Proof: Theorem 3.4.1 can be derived by applying Gil-Peleaz inversion as in Section 2.3.□

### 3.4.3 Large Scale Receive Antenna System

With many receiver antennas, we have SINR linearly scaled with the number of receive antennas \( N_j \):

\[
\text{SINR}_j \approx \frac{p_j R_{j,0}^{\alpha_j(\epsilon_j{-}1)} N_j}{\sigma^2 + I_{j,j} + \sum_{k=1}^k I_{k,j}}
\]

(3.4.16)

Proof: By using law of large number, let \( \mathbf{x}, \mathbf{y} \in \mathbb{C}^{M \times 1} \) be two independent vectors with distribution \( \mathcal{CN} (0, \sigma^2 I) \). Then \( \lim_{M \to \infty} \frac{\mathbf{x} \mathbf{y}^H}{M} = 0 \) and \( \lim_{M \to \infty} \frac{\mathbf{x} \mathbf{x}^H}{M} = \sigma^2 I \). We can have \( \|b_{j,0}\| \to N_{j,0} \).□

The framework to compute SINR coverage stays the same as in (3.4.14) but replaces \( \tilde{Q}_j(x) \) with:

\[
\tilde{Q}_j(x) = \int \exp \left(-N_j \xi^{\alpha_j(\epsilon_j{-}1)} \right) \tilde{f}_{R_j}(\xi) d\xi
\]

(3.4.17)

### 3.5 Numerical Results

We validate in this section the mathematical frameworks against Monte Carlo simulations, and study the impact of different uplink cellular setups. The simulations are done by
generating PPP located BSs per tier and MTs in a 2D plane and then doing association with specific scheme discussed in (3.4.2). To guarantee saturation, a MT near origin is selected and its associated BS is defined as the probe BS. The SINR coverage probability and average rate are simulated for the probe link.

Fig 3.1 and 3.2 validate the coverage probability framework for various cellular uplink setups and a good accuracy can be found. More specially, Fig 3.1 studies coverage performance in single tier cellular uplink and it confirms the intuition that more receive antenna leads to better coverage. Fig 3.2 sustains the general framework in Theorem 3.4.1.

Fig 3.3 validates the average rate frameworks for various cellular uplink setups and illustrates the trends of average rate as a function of path loss exponent and fractional factor. It supports the intuition that average increases by increasing path loss exponent. The effect of fractional factor, on the other hand, is complicated. A detailed discussion can be found in [3.2][3.3]. as the fractional factor increases, the rate decreases due to the loss in rate for some users whose transmit power is reduced, which is not overcome on average by the reduction in interference and increased rate for other users, especially those near the cell-edge.

Fig 3.4 plots the coverage probability for the scaled SINR \( \frac{1}{N_r} \text{SINR} \) and confirms the analysis in Section 3.4.3 that output SINR at the MRC receiver is linearly scaled with the number of receive antennas \( N_r \).

![Figure 3.1](image.png)

Figure 3.1 \( P_{\text{cov}} (\gamma) \) of a single tier cellular uplink against threshold. Setup: \( \lambda_i = 10^{-5} \) and \( \alpha_i = 6 \), \( P_M = 1 \text{Watt} \), \( p_i = -60 \text{dBm} \), \( \varepsilon_i = 0.9 \), \( \sigma^2 = 0 \text{dB} \).
Figure 3.2 $P_{\text{cov}}(\gamma)$ of a two tier cellular uplink against threshold. Setup: $\lambda_1 = 2 \times 10^{-6}$, $\lambda_2 = 10^{-5}$ and $\alpha_1 = 4$, $\alpha_2 = 5$, $P_{\text{MT}} = 0.4\text{Watt}$, $p_1 = p_2 = -60\text{dBm}$, $\epsilon_1 = \epsilon_2 = 0.9$, $T_1 / T_2 = 5$, $\sigma^2 = -20\text{dB}$.

Figure 3.3 Average rate in interference limited regime of a single tier cellular uplink against fractional factor. Setup: $\lambda_1 = 2 \times 10^{-6}$, $P_{\text{MT}} \to \infty$, $p_1 = -60\text{dBm}$. 
Figure 3.4 Scaled SINR coverage of a single tier cellular uplink with massive receive antennas against threshold. Setup: $\lambda_i = 2 \times 10^{-6}$ and $\alpha_i = 4$, $P_{MT} \to \infty$, $p_i = -60dBm$, $\varepsilon_i = 0.9$, $\sigma^2 = 10^{-12}Watt$. 
4. Performance Evaluation of Relay-Aided Downlink Cellular Networks

4.1 Performance evaluation of cellular networks with fixed relays

4.1.1 Introduction

Relay-aided wireless networks have been an active field of research for the last few years in both academia and industry[4.1]. Furthermore, the Third Generation Partnership Project's Long Term Evolution-Advanced (3GPP LTE-A) is considering relay-aided architectures for cost-effective throughput enhancement, coverage extension and energy consumption reduction[4.2], [4.3]. Among the many cooperative transmission protocols, Amplify-and-Forward (AF-) based relaying is considered to provide a good trade-off between implementation complexity, cost and achievable performance [4.4]. As such, its end-to-end error probability and achievable diversity have been studied extensively in the last years[4.5].

The available literature on the mathematical performance evaluation of AF-based relay-aided wireless networks is vast. In particular, the end-to-end error probability and achievable diversity over different fading channels have been studied in noise-limited wireless networks [4.5]-[4.11] as well as in the presence of additive noise and interference [4.12]-[4.30]. As far as the latter scenario is concerned, the available frameworks usually rely on the assumptions that either a finite number of interferers or a single dominant interferer [2.21], [2.26] is available in the network, that the locations of the interferers are fixed and known a priori, as well as that the interference follows a Gaussian distribution [2.20]. Furthermore, the diversity combiners at the destination are usually assumed to be oblivious to the interference statistics and distribution, which results in a low-complexity implementation but in sub-optimal performance. Examples of papers studying more advanced demodulators are [2.30]-[2.36]. However, they do not consider relay-aided transmissions.

These assumptions may be appropriate for modeling and studying classical communications networks, e.g., carefully planned macro cellular systems. However, they may be less appropriate to the analysis of emerging communications systems, such as heterogeneous cellular networks [4.38]-[4.40], where, e.g., cognitive radios [4.41] and closed-access femto base stations [4.22] are randomly overlaid within the macro cells. In such a scenario, both cognitive radios and closed-access femto base stations act as interferers to the macro cell users, whose number and locations are random and hence unknown a priori. Accordingly, none of the interferers can be considered dominant compared to the others and the aggregate interference largely deviates from a Gaussian distribution [4.43]-[4.47]. Motivated by these considerations, a few papers have recently studied the performance of relay-aided
wireless networks in the presence of random interference [4.48]-[4.54]. In [4.49], the outage of optimal, maximal ratio and selection combining is investigated. In [4.51], AF-based dual-hop relaying is studied without diversity combining at the destination. In [4.52], the achievable spatial-contention diversity order of cooperative relaying is computed. In [4.53] and [4.54], error probability and diversity order of multi-hop relaying are studied in interference-limited networks without diversity combining at the destination.

In this section, we study AF-based dual-hop cooperative protocols in the presence of Nakagami-m fading, additive noise at the relay, as well as additive noise and symmetric alpha-stable interference at the destination. Compared to [4.48]-[4.54], the study in the section is different in many aspects: i) two interference scenarios are investigated, which arise, e.g., when either the same or different interferers are active during the broadcast and relaying phases; ii) a Maximal Ratio Combining (MRC) and Selection Combining (SC) demodulators at the destination are analyzed. We provide closed-form expressions of the end-to-end Moment Generating Function (MGF) and study the achievable diversity for all considered setups. Four main takeaway messages emerge from the mathematical frameworks: 1) the diversity order depends on the amplitude path-loss exponent \( b \) of the interfering network; 2) under the assumption that the transmit-powers of cooperative and interfering networks are independent, the diversity order is equal to \( \frac{1}{b} \).

The following notation is used throughout this section: \( \mathbb{N}^+ \) is the set of positive natural numbers. \( \text{GF}(M) \) denotes a Galois Field (GF) of size \( M \). \( j = \sqrt{-1} \) is the imaginary unit. \( (\cdot)^* \) is the complex conjugate operator. \( \text{Re}\{\cdot\} \) is the real part operator. \( \overset{d}{=} \) denotes an equality in distribution. \( \mathcal{CN} (\mu, \sigma^2) \) is a complex Gaussian Random Variable (RV) with mean \( \mu \) and variance \( \sigma^2 \). \( \mathcal{CN}_A (\cdot) \) is a complex Gaussian RV conditioned upon the RV \( A \). \( \text{E}\{\cdot\} \) is the expectation operator. \( M_X(s) = \text{E}\{\exp\{-sX\}\} \) is the MGF of RV \( X \). \( F_X(\xi) = \text{Pr}\{X \leq \xi\} \) is the Cumulative Distribution Function (CDF) of RV \( X \). \( F_X^{(c)}(\xi) = 1 - F_X(\xi) \) is the Complementary Cumulative Distribution Function (CCDF) of RV \( X \). \( f_X(\cdot) \) is the Probability Density Function (PDF) of RV \( X \). \( \binom{z}{x} \) is the binomial coefficient. \( \Gamma(x) = \int_{0}^{\infty} t^{x-1} \exp\{-t\} dt \) is the Gamma function. \( \Gamma(z,x) = \int_{x}^{\infty} t^{z-1} \exp\{-t\} dt \) is the upper-incomplete Gamma function.
\[ \gamma(z, x) = \int_0^x t^{z-1} \exp\{-t\} \, dt \] is the lower-incomplete Gamma function.

\[ B(x, y) = \frac{\Gamma(x) \Gamma(y)}{\Gamma(x + y)} \] is the Beta function. \( (x) = \frac{\Gamma(x + y)}{\Gamma(x)} \) is the Pochhammer symbol. \( x! = \Gamma(x + 1) \) is the factorial operator with \( x \) be a natural number.

\[ E_n(x) = \int_1^{\infty} t^{-n} \exp\{-xt\} \, dt \] is the generalized exponential integral function. \( _1 F_1(\cdot; \cdot; \cdot) \) is the Kummer confluent hypergeometric function \[4.55\]. \( G_{\nu,q}^{m,n} \left( \begin{array}{c} \mathbf{a} \\ \mathbf{b} \end{array} \right) \) is the Meijer G-function[4.56]. \[ \Delta(x, y) = \left( y / x, (y + 1) / x, \ldots, (y + x - 1) / x \right) \] with \( x \in \mathbb{R}^+ \) and \( y \in \mathbb{R}^+ \).

\[ Y(0) = 1 \] and \[ Y(g) = \prod_{f=0}^{g-1}(b_f^{-1} - f) \] for \( g \in \mathbb{R}^+ \).

### 4.1.2 System Model

The dual-hop network topology sketched in Fig. 4.1 is studied, which corresponds to a typical scenario considered by the IEEE 802.16 working group for relay-aided communications \[4.2\].

A three-node network is considered, where a tower-mounted source (\( S \)) communicates with the intended destination (\( D \)) located at the street level, with the aid of a fixed relay station (\( R \)) deployed on the rooftop of a building. The destination receives two copies of the same information-bearing signal from the source and the relay. The destination is surrounded by randomly distributed co-channel interferers, whose spatial locations are unknown a priori and can vary over the whole bi-dimensional plane. The locations of source, relay and destination, on the other hand, are assumed to be fixed and known. Since the relay is located on the rooftop and the randomly-distributed interferers are located at the street level (see Fig. 4.1), the relay is assumed to be noise-limited and the destination is subject to both noise and co-channel interference. This scenario is similar to \[4.12\]. In our work, however, the interferers have random spatial locations. Possible application scenarios of this network deployment to heterogeneous cellular networks are described in \[4.47\].
Fig. 4.1: System model

The transmission of $S$ and $R$ occur in two orthogonal time-slots. In the first time-slot, $S$ broadcasts its data to $R$ and $D$. Let $\mu_s \in \mathbb{GF}(M)$ be the symbol transmitted by $S$. The signals received at $R$ and $D$ can be written as follows:

$$
\begin{align*}
    y_{SR} &= \sqrt{E_s} h_{SR} x_s + n_{SR}, \\
    y_{SD} &= \sqrt{E_s} h_{SD} x_s + n_{SD} + i_t,
\end{align*}
$$

(4.1)

where: i) $x_s = \varphi_M(\mu_s)$ is the $M$-ary modulated symbol emitted by $S$; ii) $\varphi_M(\cdot)$ is the modulation mapping function; iii) $E_s$ is the average symbol energy of $S$; iv) $n_{XY}$ is the complex Additive White Gaussian Noise (AWGN) at the input of node $Y$ and related to the transmission from node $X$. The AWGN is independent and identically distributed (i.i.d.) with zero mean and variance $N_0/2$ per real dimension, i.e., $n_{XY} \sim \mathcal{CN}(0,N_0)$; v) $i_t$ is the aggregate interference at $D$ in the first time-slot, which is described in Section 4.2.2; vi) $h_{XY}$ is the fading gain from node $X$ to node $Y$. In particular, $h_{XY} = (\kappa_{XY} / d_{XY}^{\beta_{XY}}) \alpha_{XY} \exp(j\phi_{XY})$,

where: $\kappa_{XY}$ is a propagation-dependent constant; $d_{XY}$ is the nodes’ distance; $b_{XY}$ is the amplitude path-loss exponent; $\alpha_{XY}$ is the fading envelope, which is assumed to follow a Nakagami-m distribution having parameters $\left(m_{XY}, \Omega_{XY}\right)$. For mathematical tractability, $m_{XY}$ is assumed to be an integer. It is known that, in general, $m_{XY}$ may take non-integer values [4.10],[4.58]. The analysis of these scenarios, however, is postponed to future research. The Nakagami-m fading model is considered since it is useful for modeling both line-of-sight and...
non-line-of-sight propagation conditions [4.57]; and \( \phi_{XY} \) is the channel phase, which is assumed to follow a uniform distribution in \( [0, 2\pi) \).

In the second time-slot, \( R \) uses the AF protocol for relaying \( y_{SR} \) to \( D \). The signal received at \( D \) can be formulated as \( y_{RD} = \sqrt{E_R h_{RD}} (G_{AF} y_{SR}) + n_{RD} + i_{T_2} \), where: i) \( E_R \) is the average symbol energy of \( R \); ii) \( G_{AF} = 1 / \left( \sqrt{E_s h_{SR}} \right) \) is the AF relaying gain, by assuming an ideal Channel State Information (CSI-) assisted protocol[4.9]. More accurate harmonic mean approximations for AF-based relaying have recently been proposed in the literature [4.59]. For mathematical tractability, however, they are not considered in this paper; and iii) \( i_{T_2} \) is the aggregate interference at \( D \) in the second time-slot, which is described in Section 4.2.2.

### 4.1.3 Network Interference Model

For ease of notation but without loss of generality, \( D \) is located at the origin of the bi-dimensional plane where the interferers are randomly distributed e.g., the street level of Fig. 4.1. The interferers are modeled as points of a Poisson Point Process (PPP) of fixed density \( \lambda \), similar to [4.43]-[4.47]. \( \lambda \) is the average number of interferers per square meter and it is measured in “interferers/m\(^2\)”. By relying on a statistical-physical generation and propagation mechanism of the network interference, [4.43]-[4.47] have proved that \( i_{T_1} \) and \( i_{T_2} \) follow a Symmetric Alpha-Stable (S\( \alpha \)S) distribution [4.60]. With the aid of the compound Gaussian representation of [4.47], they can be formulated as \( i_{T_2} = \sqrt{E_I} \sqrt{B_Z G_Z} \), where: i) \( E_I \) is the average symbol energy of the interferers; ii) \( Z = 1, 2 \) identifies the time-slot; iii) \( B_Z \) is a Stable RV totally skewed to the right, which is denoted by \( B_Z \sim S_{\alpha b} \left( 1 / b_I, 1, 1, 0, 0 \right) \) with \( b_I > 1 \) being the amplitude path-loss exponent of the interferers-to-\( D \) links [4.60]. The RV \( B_Z \) is characterized by its MGF, which is \( M_{B_Z} (s) = \exp \left( -s^{1/b_I} \right) \); and iv) \( G_Z \sim \text{CN} \left( 0, \sigma_{G_Z}^2 \right) \).

The RVs \( B_Z \) and \( G_Z \) are independent [4.60]. The variance \( \sigma_{G_Z}^2 \) depends on the propagation parameters of the interferers-to-\( D \) links (i.e., fading distribution, \( b_f \)), \( \lambda \) and the modulation scheme of the interferers. Closed-form expressions of \( \sigma_{G_Z}^2 \) are available in [4.47], to which the reader is referred for further information. As an example, let \( (\Theta_{ID}, \Omega_{ID}) \) be the parameters of the Nakagami-\( m \) fading of the i.i.d. interfering channels and let the modulation used by the
interferers be the Multilevel Phase Shift Keying (MPSK). In this case, \( \sigma_{b_i}^2 = 4(\lambda \pi K_i K_1)^{b_i} \) [4.47], where:

\[
K_1 = \frac{2}{\pi} \quad \text{if} \quad b_i = 2 \quad \text{and} \quad K_i = \frac{1 - 2/b_i}{\Gamma(2 - 1/b_i) \cos(\pi/b_i)} \quad \text{if} \quad b_i \neq 2
\]

\[
K_2 = \frac{\Gamma(m_{ID} + 1/b_i)}{\Gamma(m_{ID}) m_{ID}^{1/b_i}} \Omega_{\text{ID}b}^{1/b_i} \left( \frac{\Gamma(1/b_i - 1/2)}{2\sqrt{\pi} \Gamma(1/b_i)} - \frac{\Gamma(2 - 1/b_i) \Gamma(1/b_i - 1/2)}{2\sqrt{\pi} \Gamma(1 - 1/b_i) \Gamma(1 + 1/b_i)} \right) \tag{4.2}
\]

Since the communication occurs in two time-slots, the temporal correlation properties of \( i_{T_1} \) and \( i_{T_2} \) have to be characterized. Two practical scenarios are considered, which take into account the different session lifetimes of the dual-hop relaying protocol and of the interferers: 1) the quasi-static scenario and 2) the fast-varying scenario. In the former case, the interferers are assumed to have a longer session lifetime than the duration of the cooperation phase, which accounts for broadcast and relaying. Thus, it is reasonable to assume that the same interferers are active in both time-slots and that they do not change their locations. Also, the fading envelopes and phases are assumed not to change in the two time-slots. Hence, \( i_{T_1} \) and \( i_{T_2} \) are correlated and \( i_{T_1} = i_{T_2} \). In the latter case, the interferers are assumed to have a shorter session lifetime than the duration of the cooperation phase. More precisely, disjoint sets of interferers are active during broadcast and relaying phases. Owing to the different active interferers and to their different locations, the associated fading envelopes and phases are assumed to be independent in the two time-slots. Hence, \( i_{T_1} \) and \( i_{T_2} \) can be assumed to be independent. The quasi-static scenario may occur when the relaying phase is scheduled right after the broadcast phase. The fast-varying scenario may occur when broadcast and relaying phases are delayed. It is worth mentioning that the impact of interference correlation on the performance of wireless networks is receiving an upsurge of research interest [4.46],[4.47],[4.61]-[4.64]. The analysis and comparison of quasi-static and fast-varying scenarios provide a contribution to this field of research.

### 4.1.4 Problem Statement

Let the system model of Section 4.2.2, the signals received at \( D \) can be formulated as follows:

\[
\begin{align*}
y_{SD} & = \sqrt{E_s} h_{SD} x_s + n_{SD} + \sqrt{E_t} \sqrt{B_G} (a) = \sqrt{E_s} h_{SD} x_s + n_{SD} \\
y_{RD} & = \sqrt{E_k} h_{RD} x_s + (n_{RD} + \sqrt{E_k} h_{RD} (\sqrt{E_s} h_{SR})^{-1} n_{SR}) + \sqrt{E_t} \sqrt{B_G} (b) = \sqrt{E_k} h_{RD} x_s + n_{SRD}
\end{align*}
\tag{4.3}
\]
where $E_{XY} = E_X\left(\kappa_{XY} / d_ {XY}^2\right)^2 \, (X = \{S, R\}, \, Y = \{R, D\})$ and (a), (b) follow by introducing:

\[
\begin{align*}
\eta_{SD} &= n_{SD} + \sqrt{E_{1}B_{1}}G_{1} \sim \text{CN}_{|r_{k}}(0, N_{0} + E_{1}B_{1}\sigma_{G_{1}}^2) \\
\eta_{SRD} &= \left(n_{RD} + \sqrt{E_{2}h_{RD}\left(\sqrt{E_{5}h_{SR}}\right)^{-1} n_{SR}} + \sqrt{E_{1}\sqrt{B_{2}}G_{2} \sim \text{CN}_{|r_{k-x}}(0, N_{0} + N_{0}E_{RD}\alpha_{RD}^2 \left(\tilde{E}_{SR}\alpha_{SR}^2\right)^{-1} + E_{1}B_{2}\sigma_{G_{2}}^2}\right)
\end{align*}
\]

(4.4)

where $B = B_{1} = B_{2}, \, G = G_{1} = G_{2}$ in the quasi-static scenario, while $(B_{1}, B_{2}), \, (G_{1}, G_{2})$ are i.i.d. in the fast-varying scenario. Their distributions are determined by $b_{i}, \, \sigma_{G_{i}}^2 = \sigma_{G_{2}}^2 = \sigma_{G_{2}}^2$.

The objective is to compute the end-to-end error probability and to study the diversity order of the dual-branch distributed network of (4.1). These performance metrics are computed by averaging over all possible network deployments of the interferers, according to the definition of "spatial average" given in [4.47]. From (4.1), this implies the need of computing the expectations over the RVs $B_{2}$ and $G_{2}$. Since $y_{SD}$ and $y_{RD}$ are complex Gaussian RVs by conditioning upon $h_{XY} \, , \, B_{1} \, , \, B_{2}$, a two-step methodology is proposed: 1) first, the performance metrics are computed by averaging over the distribution of the AWGNs and the RVs $G_{2}$; and 2) then, the expectation over the other RVs is computed. This approach is convenient because the error probability of diversity systems in AWGN (first step) has been widely studied.

By using this two-step approach, the end-to-end error probability of general bi-dimensional modulations can be formulated as the linear combination of integrals like [4.47]:

\[
J(\sigma, \psi, \chi) = \frac{1}{\pi} \int_{-\infty}^{\infty} E_{h, B} \left\{ \exp\left( -\chi^2 \frac{\sin^2(\psi)}{\sin^2(\theta)} \text{SINR}(h, B) \right) \right\} d\theta
\]

(4.5)

where: i) $h$ and $B$ are short-hands collecting the RVs $(h_{SR}, h_{SD}, h_{RD})$ and $(B_{1}, B_{2})$; ii) $\text{SINR}(h, B)$ is the end-to-end Signal-to-Interference-plus-Noise-Ratio (SINR), which depends on the demodulator used at $D$ and is conditioned upon $h \, , \, B$; iii) $M_{\text{SINR}}(s) = E_{h, B} \{ \exp\{-s\text{SINR}(h, B)\} \}$; and iv) $(\sigma, \psi, \chi)$ are modulation-dependent parameters [2.10]. For example, $(\sigma, \psi, \chi) = (\pi(M-1) / M, \pi / M, 1)$ for MPSK modulation [4.57]. The equalities in (a) and (b) correspond to first and second step of the two-step
approach described above, respectively. The computation of (4.2) may be avoided using approximations [4.65].

The application of (4.2) requires a formal definition of $\text{SINR}(\cdot, \cdot)$, which depends on the demodulator used at $D$. The decision statistic of a general demodulator can be formulated as:

\[
\Lambda(\bar{\mu}_s) \propto D_1(E_s, E_R, h)|\Delta_s|^2 + 2\text{Re}\{D_2(E_s, E_R, h, n)\Delta_s^*\} + 2\text{Re}\{D_3(E_s, E_R, E_I, h, G, B)\Delta_s^*\}
\]

(4.6)

where $\bar{\mu}_s$ is the hypothesis of $\mu_s$, $\Delta_s = \varphi_M(\bar{\mu}_s) - \varphi_M(\mu_s)$, $n$ and $G$ are short-hands collecting the RVs $(n_{SR}, n_{SD}, n_{RD})$ and $(G, G)$, and $D_1(\cdot), D_2(\cdot), D_3(\cdot)$ are related to useful, noise and interference terms, respectively. The general formulation in (4.6) applies to all the demodulators of this paper. From (4.6), the $\text{SINR}(\cdot, \cdot)$ can be formulated as:

\[
\text{SINR}(h, B) = \frac{|D_1(E_s, E_R, h)|^2}{2E_n\{(\text{Re}\{D_2(E_s, E_R, h, n)\})^2\} + 2E_G\{(\text{Re}\{D_3(E_s, E_R, E_I, h, G, B)\})^2\}}
\]

(4.7)

### 4.1.5 Diversity Order in Wireless Networks with Noise and Interference

Let $E_T = E_s + E_R$. In noise-limited networks, the diversity order is defined as the slope of the end-to-end error probability in (4.5) as a function of $E_T / N_0 \nrightarrow 1$ in a log-log scale [4.66]. In interference-limited networks, on the other hand, the diversity order is defined as the slope of the end-to-end error probability in (4.5) as a function of $E_T / E_I \nrightarrow 1$ in a log-log scale [4.62].

In wireless networks with noise and interference, it is relevant to study the asymptotic behavior of the end-to-end error probability in (4.5) as a function of both $E_T / N_0 \nrightarrow 1$ and $E_T / E_I \nrightarrow 1$. In this paper, two case studies are of interest [4.44]: 1) the homogeneous scenario, i.e., $E_T / N_0 = \tau_1(E_T / N_0)$ with $\tau_1 > 0$ and 2) the heterogeneous scenario, i.e., $E_T / N_0$ and $E_I / N_0$ are independent of each other. The interested reader is referred to [4.44] for further details. As for the heterogeneous scenario, two situations are worth being studied: 2a) the diversity order as a function of $E_T / N_0 \nrightarrow 1$ when $E_I / N_0$ is kept constant and 2b) the diversity order as a function of $E_T / E_I \nrightarrow 1$ when either $E_I / N_0$ or $E_T / N_0$ are kept constant, as $E_T / E_I$ can be increased by either increasing $E_T$ and keeping $E_I$ constant or by
decreasing \( E_f \) and keeping \( E_f \) constant, respectively. Throughout this paper, \( N_0 \) is assumed to be constant for all the analyzed case studies. In practice, scenarios 2a) and 2b) are useful for comparing the system setup under analysis against noise- and interference-limited networks, where the diversity order is computed as a function of \( E_f / N_0 \) and \( E_f / E_i \), respectively. For example, scenario 2a) allow us to readily assess the diversity order loss due to the aggregate interference compared to interference-free wireless networks, e.g., [4.12], [4.13], [4.20], [4.21] and [4.44]. A similar comment applies to scenario 2b) as far as interference-limited networks are concerned, e.g., [4.62]. However, scenario 2b) reduces either to the noise-limited setup or to scenario 2a). In particular: i) if \( E_f \) is kept constant but \( E_i \) decreases, the system reduces to the noise-limited case as the interference power is negligible compared to the noise power. This scenario is of no interest in this paper, as it has been studied extensively in the literature already [4.5]-[4.11]; and ii) if \( E_i \) is kept constant but \( E_f \) increases, the diversity analysis is the same as 2a). This implies that the diversity order of case study 2b) can be obtained from the analysis of case study 2a) by simply replacing \( E_f / N_0 \) with \( E_f / E_i \). A similar comment applies if \( E_f / N_0 \triangleq 1 \) and \( E_f / E_i \triangleq 1 \), i.e., \( E_f \) increases and \( E_i \) decreases at the same time. Thus, only case studies 1) and 2a) are analyzed in this paper.  

Similar to [4.66], the diversity order can be obtained from the asymptotic behavior of \( M_{\text{SINR}}(\cdot) \) in (5). In fact, \( M_{\text{SINR}}(\cdot) \) is a function of \( E_f / N_0 \) and \( E_f / E_i \). Let \( \Re \) be the ratio of interest, e.g., \( \Re = E_f / N_0 \) or \( \Re = E_f / E_i \). Then, the following definition of diversity holds.

**Definition 1:** Let \( M_{\text{SINR}}(s)|_{\Re=1} \rightarrow K\Re^{-d_o}Z(s) \), where \( K > 0 \) and \( Z(\cdot) \) is independent of \( \Re \). From (5), the asymptotic error probability can be formulated as the linear combination of:

\[
J(\sigma, \psi, \chi)|_{\Re=1} \rightarrow \frac{1}{\pi} \int_0^\pi K\Re^{-d_o}Z \left( \frac{\chi^2 \sin^2(\psi)}{\sin^2(\theta)} \right) d\theta = \Re^{-d_o} \left( \frac{K}{\pi} \int_0^\pi Z \left( \frac{\chi^2 \sin^2(\psi)}{\sin^2(\theta)} \right) d\theta \right) \quad (4.8)
\]

Then, the diversity order as a function of \( \Re \) is defined to be \( d_o \).
From (5) and Definition 1, error probability and diversity order can be obtained from $M_{\text{SINR}}(\cdot)$. In the next sections, $M_{\text{SINR}}(\cdot)$ is computed in closed-form for different demodulators at $D$.

### 4.1.6 End-to-end error probability of dual-hop cooperative relaying using a maximum ratio combining receiver

Consider a MRC demodulator, which has perfect knowledge of the CSI, i.e., $h_{XY}$, of the three-node cooperative network but is oblivious to the network interference. This corresponds to the optimal diversity combiner without network interference [4.57] ($\bar{x}_S = \varphi_M(\tilde{\mu}_S)$):

$$\hat{\mu}_S = \arg\min_{\tilde{\mu}_S \in \mathbb{GF}(M)} \left\{ \Lambda(\tilde{\mu}_S) = \left( N_0 \right)^{-1} \left| y_{SD} - \sqrt{E_s} h_{SD} \bar{x}_S \right|^2 + \left( N_0 + N_0 \bar{E}_{RD} \alpha_{RD}^2 \left( \bar{E}_{SR} \alpha_{SR}^2 \right)^{-1} \right)^{-1} \left| y_{RD} - \sqrt{E_r} h_{RD} \bar{x}_S \right|^2 \right\}$$

(4.9)

**Quasi-Static Interference Scenario**

Considering the conditional-AWGN representation, the SINR($\cdot$) in (4.7) is:

$$\text{SINR}(h,B) \left( \gamma_{SD} + \frac{\gamma_{SR} \gamma_{RD}}{\gamma_{SR} + \gamma_{RD}} \right)^2 \left( \frac{\gamma_{SD}}{\gamma_{SR} + \gamma_{RD}} + \frac{\gamma_{SR} \gamma_{RD}}{\gamma_{SR} + \gamma_{RD}} \right) + \gamma_{SD} \gamma_{B} + \frac{\gamma_{SR} \gamma_{RD}}{\gamma_{SR} + \gamma_{RD}} \gamma_{B} \cos(\Delta \phi)$$

where $\gamma_{XY} = \left( \bar{E}_{XY} / N_0 \right) \alpha_{XY}^2$, $\gamma_{B} = \left( \bar{E}_{I} / N_0 \right) \beta \sigma_G^2$, $\Delta \phi = \phi_{RD} - \phi_{SD}$. The SINR in (4.10) is exact but mathematically intractable. Lemma 1 overcomes this issue with the aid of equivalent channel.

**Lemma 4.1.** Let the equivalent channel model, the optimal interference-oblivious demodulator and its equivalent output SINR are:

$$\hat{\mu}_S = \arg\min_{\tilde{\mu}_S \in \mathbb{GF}(M)} \left\{ \Lambda^{(eq)}(\tilde{\mu}_S) = \left| y_{SD} - \sqrt{E_s} h_{SD} \bar{x}_S \right|^2 + \left| y_{RD} - \sqrt{E_r} h_{RD} \bar{x}_S \right|^2 \right\}$$

$$\text{SINR}^{(eq)}(h,B) \approx \left( 1 + \gamma_{B} \right)^{-1} \left( \gamma_{SD} + \min \{ \gamma_{SR}, \gamma_{RD} \} \right)$$

(4.11)

From (4.11), the MGF of the SINR in (4.10) is computed in Proposition 4.1. According to Definition 1, the asymptotic MGF is computed in Propositions 4.2, 4.3 for case studies 1) and 2a) discussed in Section II.D. Finally, the diversity order of both case studies is analyzed in Proposition 4.4.
Proposition 4.1: Let the system model of Section II. The MGF of (4.10) is

\[ M_{\text{SINR}}(s) = M_{\text{SINR}(s)}(s), \]

where

\[ M_{\text{SINR}(s)}(s) = \sum_{i=0}^{m_{\text{SD}}} \sum_{k=0}^{m_{\text{RD}}} Z_{i,k}^{(1)} \left( \frac{s^{m_{\text{SR}}}}{\gamma_{\text{SD}}} \right)^{-k} Y_k(s) \left( \frac{s^{m_{\text{SR}}}}{\gamma_{\text{SR}}} \right) - \sum_{i=0}^{m_{\text{SD}}} \sum_{k=0}^{m_{\text{RD}}} Z_{i,k}^{(2)} \left( \frac{s^{m_{\text{SR}}}}{\gamma_{\text{SD}}} + \frac{s^{m_{\text{RD}}}}{\gamma_{\text{RD}}} \right)^{-k} Y_k(s) \left( \frac{s^{m_{\text{SR}}}}{\gamma_{\text{SR}}} + \frac{s^{m_{\text{RD}}}}{\gamma_{\text{RD}}} \right) \]

\[ + \sum_{i=0}^{m_{\text{SD}}} \sum_{k=0}^{m_{\text{RD}}} Z_{i,k}^{(3)} \left( \frac{s^{m_{\text{SD}}}}{\gamma_{\text{SD}}} \right)^{-k} Y_k(s) \left( \frac{s^{m_{\text{SR}}}}{\gamma_{\text{SR}}} \right) - \sum_{i=0}^{m_{\text{SD}}} \sum_{k=0}^{m_{\text{RD}}} Z_{i,k}^{(4)} \left( \frac{s^{m_{\text{SD}}}}{\gamma_{\text{SD}}} + \frac{s^{m_{\text{RD}}}}{\gamma_{\text{RD}}} \right)^{-k} Y_k(s) \left( \frac{s^{m_{\text{SR}}}}{\gamma_{\text{SR}}} + \frac{s^{m_{\text{RD}}}}{\gamma_{\text{RD}}} \right) \]

\[ \text{(4.12)} \]

\[ Z_{i,k}^{(1)} = \frac{m_{\text{SD}}}{\gamma_{\text{SD}}} \Gamma(m_{\text{SD}}) \left( \frac{s^{m_{\text{SR}}}}{\gamma_{\text{SR}}} \right)^{-i!} \left( \frac{s^{m_{\text{RD}}}}{\gamma_{\text{RD}}} \right)^{-i!} T \left( m_{\text{SD}} - 1, i + m_{\text{SR}} - 1, \delta_{\text{SRD}} \right) \]

\[ Z_{i,k}^{(2)} = \frac{m_{\text{RD}}}{\gamma_{\text{RD}}} \Gamma(m_{\text{RD}}) \left( \frac{s^{m_{\text{SR}}}}{\gamma_{\text{SR}}} \right)^{-i!} \left( \frac{s^{m_{\text{RD}}}}{\gamma_{\text{RD}}} \right)^{-i!} T \left( m_{\text{SD}} - 1, i + m_{\text{SR}} - 1, \delta_{\text{SRD}} \right) \]

\[ Z_{i,k}^{(3)} = \frac{m_{\text{SD}}}{\gamma_{\text{SD}}} \Gamma(m_{\text{SD}}) \left( \frac{s^{m_{\text{SR}}}}{\gamma_{\text{SR}}} \right)^{-i!} \left( \frac{s^{m_{\text{RD}}}}{\gamma_{\text{RD}}} \right)^{-i!} T \left( m_{\text{SD}} - 1, i + m_{\text{RD}} - 1, \delta_{\text{SRD}} \right) \]

\[ Z_{i,k}^{(4)} = \frac{m_{\text{RD}}}{\gamma_{\text{RD}}} \Gamma(m_{\text{RD}}) \left( \frac{s^{m_{\text{SR}}}}{\gamma_{\text{SR}}} \right)^{-i!} \left( \frac{s^{m_{\text{RD}}}}{\gamma_{\text{RD}}} \right)^{-i!} T \left( m_{\text{SD}} - 1, i + m_{\text{RD}} - 1, \delta_{\text{SRD}} \right) \]

\[ \text{(4.13)} \]

\[ Y_k(s, \xi) = \Gamma(k + 1) - \Gamma(k + 1)(2\pi)^{-(i-1)/2-(n-1)/2} \sqrt{I_1} \]

\[ \times \sum_{g=0}^{k} \sum_{f=0}^{g} \left( \frac{g}{f} \right)^{(s-f)/2} \left( \frac{s+f}{s+\xi} \right)^{-(i+1)/2} \Omega_k \left( \frac{E_x \xi^2}{N_0} \right)^{i/2} \left( \frac{l_i}{\xi^{i+1}} \right)^{(i+1)/2} \Delta(r_i, 0) \Delta(r_i, f-g) \]

\[ \text{(4.14)} \]

\[ T(x, y, z) = B(1 + x + y)(x + y) \left( \frac{y}{x} \right)^{(x-y-1) \gamma_x} \left( \frac{y}{z} \right)^{(y-z) \gamma_y} P_1(x, y, z, w) \]

\[ = (-y_x)^{(x-y-1) \gamma_x} (-z_x)^{(y-z) \gamma_y} P_2(x, y, z, w) \]

\[ \text{(4.15)} \]

where: i) \( 1/b_i = r_i/l_i \) with \( r_i \), \( l_i \) being positive integers; ii) \( \delta_{\text{SRD}} = -m_{\text{SD}}/\gamma_{\text{SD}} + m_{\text{SR}}/\gamma_{\text{SR}} + m_{\text{RD}}/\gamma_{\text{RD}} \); and iii) \( \gamma_{XY} = (E_{XY}/N_0) \Omega_{XY} \).

Proposition 4.2: Let the same assumptions as in Proposition 4.1. Let \( E_{S} = K_T E_{T} \), \( E_{R} = (1-K_T) E_{T} \) and \( E_{T} = E_{S} + E_{R} \) with \( 0 < K_T \leq 1 \). Let \( E_I / N_0 = \tau I (E_{T} / N_0) \). If \( E_{T} / N_0 \leq 1 \), the MGF of the SINR in (4.10) is approximated by (4.12) obtained by replacing \( Y_k(\cdot, \cdot) \) of (4.14) with \( \bar{Y}_k(\cdot, \cdot) \):
\[
\tilde{Y}_k(s, \xi) = \Gamma(k + 1) - \Gamma(k + 1)(2\pi)^{(1-i+\tau)/2} \sqrt{\frac{2^\eta}{\eta\Gamma\eta}} \sum_{g=0}^{k} (-1)^g \left( \frac{\eta\Gamma\eta}{g!} \right) \left( \frac{r_g \xi}{s^\eta} \right)^g \left( \frac{1}{\eta\Gamma\eta} \right) \Delta(t_l, 0) \left( \Delta(t_l, 0) \right)
\]

(4.16)

and \( \tilde{f}_{XY} \) with \( \nu_{XY} \), where \( \nu_{XY} = K_T \left( K_{xy} / d_{xy}^{b_0} \right)^2 \Omega_{xy} \) and \( \nu_{RD} = (1 - K_T) \left( K_{RD} / d_{RD}^{b_0} \right)^2 \Omega_{RD} \).

**Proposition 4.3:** Let the same assumptions as in Proposition 4.1. Let \( E_s = K_T E_T \), \( E_r = (1 - K_T) E_T \) and \( E_s = E_s + E_r \) with \( 0 < K_T < 1 \). Let \( E_s / N_0 = \rho_I \) be constant, as discussed above (heterogeneous case). If \( E_r / N_0 \notin 1 \), the MGF of the SINR in (4.10) is approximated by (4.12) obtained by replacing \( Y_k(\cdot, \cdot) \) of (4.14) with \( \hat{Y}_k(\cdot, \cdot) \):

\[
\hat{Y}_k(s, \xi) = \left( \frac{E_s}{N_0} \right)^{-1/b_l} \left( \rho_I \sigma_0^2 \right)^{-1/b_l} \left( \frac{1}{b_l} + 1 \right) \left( k + 1 \right)^{-1/b_l} \left( \sum_{g=0}^{k} \left( \frac{1}{b_l} \right) \gamma(g) \right) \left( \sum_{g=0}^{k} \left( \frac{1}{g!} \right) \gamma(g) \right)^{-1/b_l} \left( \sum_{g=0}^{k} \left( \frac{1}{g!} \right) \gamma(g) \right)^{-1/b_l} \left( \sum_{g=0}^{k} \left( \frac{1}{g!} \right) \gamma(g) \right)^{-1/b_l}
\]

(4.17)

\( \tilde{f}_{XY} \) with \( \nu_{XY} \) defined in Proposition 2.

**Proposition 4.4:** The diversity order of (4.9) in a quasi-static interference scenario is equal to 0 under the assumptions of Proposition 4.2 and to \( 1/b_l \) under the assumptions of Proposition 4.3.

**Remark 4.1:** From Propositions 2-4, we conclude that the diversity order is independent of the fading parameters \( m_{XY} \). Under the assumptions of Proposition 2, i.e., the homogeneous scenario of Section II.D[4.44], the error probability reaches an horizontal asymptote for high-SNR, which implies that an optimal transmit-power exists beyond which the error probability cannot be further improved. Under the assumptions of Proposition 3, i.e., the heterogeneous scenario of Section II-D[4.44], the diversity order is the same as in single-hop networks [4.47], which implies that there is no distributed diversity gain by using the interference-oblivious MRC demodulator in (4.9). A comprehensive physical justification for the fractional diversity order equal to \( 0 < 1/b_l < 1 \) is available in[4.47].

**Fast-Varying Interference Scenario**

Similar to Section 4.3.1, the SINR(\( \cdot, \cdot \)) in (4.7) can be formulated as (\( \gamma_{Bx} = (E_s / N_0) B_x \sigma_0^2 \)): 
\[
\text{SINR}(h, B) = \left( \frac{\gamma_{SD} + \gamma_{SR} \gamma_{RD}}{\gamma_{SR} + \gamma_{RD}} \right)^2 \left[ \left( \frac{\gamma_{SD} + \gamma_{SR} \gamma_{RD}}{\gamma_{SR} + \gamma_{RD}} \right) + \left( \frac{\gamma_{SD} \gamma_{Rb} + \gamma_{SR}^2 \gamma_{RD}}{(\gamma_{SR} + \gamma_{RD})^2} \gamma_{Bz} \right) \right]^{-1}
\] (4.18)

The SINR in (4.18) is mathematically intractable and Lemma 4.2 is used instead.

**Lemma 4.2:** The optimal diversity combiner in the absence of interference is still \( \Lambda^{(eq)}(\cdot) \) in (4.12). A Lower-Bound (LB) and a Upper-Bound (UB) for the output SINR are as follows:

\[
\begin{align*}
\text{SINR}^{(eq)}_{\text{LB}}(h, B) &\leq \text{SINR}^{(eq)}(h, B) \leq \text{SINR}^{(eq)}_{\text{UB}}(h, B) \\
\text{SINR}^{(eq)}_{Z}(h, B) &= (1 + K_2 \gamma_{B})^{-1} \left( \gamma_{SD} + \min \{ \gamma_{SR}, \gamma_{RD} \} \right)
\end{align*}
\] (4.19)

where \( K_{LB} = 2^{h-1} \) for the LB, \( K_{UB} = 1 \) for the UB and \( Z \in \{\text{LB, UB}\} \).

**Remark 4.2:** By comparing (4.11) and (4.19), the MGF of the SINR in (4.19) can be obtained from Propositions 4.1-4.3 by replacing \( \sigma_G^2 \) with \( K_2 \sigma_G^2 \). Hence, the diversity order in the fast-varying scenario is the same as in the quasi-static scenario, as summarized in Proposition 4.4.

**Remark 4.3:** Since \( \text{SINR}^{(eq)}_{\text{UB}}(\cdot, \cdot) \) in (4.19) is equal to \( \text{SINR}^{(eq)}(\cdot, \cdot) \) in (4.11), the error probability in the fast-varying scenario is never better than in the quasi-static scenario.

### 4.1.7 End-to-end error probability of dual-hop cooperative relaying using a selection combining receiver

Consider a SC demodulator, which has perfect knowledge of the CSI, i.e., \( h \), of the three-node cooperative network but is oblivious to the network interference. Under these assumptions, it can only choose the branch providing the highest Signal-to-Noise Ratio (SNR), which is computed by neglecting the aggregate interference. From (4.3), it can be formulated as [4.57]:

\[
\hat{\mu}_s = \arg \min_{\mu_s \in \mathbb{GF}(M)} \left\{ \Lambda^{(eq)}(\mu_s) \right\} = \begin{cases} 
\left[ \gamma_{SD} - \sqrt{E[Z \sigma_{DM}^2(\mu_s)]} \right]^2 & \text{if } \gamma_{SD} \geq \gamma_{eq} = \min \{ \gamma_{SR}, \gamma_{RD} \} \\
\left[ \gamma_{SRD} - \sqrt{E[Z \sigma_{DM}^2(\mu_s)]} \right]^2 & \text{if } \gamma_{SD} < \gamma_{eq} = \min \{ \gamma_{SR}, \gamma_{RD} \}
\end{cases}
\] (4.20)

where \( \hat{\mu}_s \) is the estimate of \( \mu_s \) and \( \gamma_{XY} = \left( \frac{E_{XY}}{N_0} \right) \alpha_{XY}^2 \).
**Quasi-Static Interference Scenario**

Consider the quasi-static scenario with \( i_t = i_z = \sqrt{E_t \sqrt{BG}} \). Lemma 4.3 provides the end-to-end error probability of (4.20), i.e., \( \text{Pr} \{ \hat{\mu}_s \neq \mu_s \} \).

**Lemma 4.3:** Let \( \mu_s \) belong to a generic bi-dimensional constellation diagram characterized by the triplet of parameters \((\sigma, \psi, \chi)\) [4.47]. The end-to-end error probability of (4.20) can be formulated as the linear combination of integrals like (4.5) with \( \gamma_B = \left( E_t / N_0 \right) B \sigma_B^2 \) and 
\[
\text{SINR}^{(eq)}(h, B) = (1 + \gamma_B)^{-1} \max \{ \gamma_{SD}, \min \{ \gamma_{SR}, \gamma_{RD} \} \}.
\]

From Lemma 4.3, error probability and diversity order can be obtained from \( M_{\text{SINR}^{(eq)}}(\cdot) \) in Definition 4.1. A closed-form expression of \( M_{\text{SINR}^{(eq)}}(\cdot) \) is given in Proposition 4.5.

**Proposition 4.5:** The MGF of \( \text{SINR}^{(eq)}(h, B) = (1 + \gamma_B)^{-1} \max \{ \gamma_{SD}, \min \{ \gamma_{SR}, \gamma_{RD} \} \} \) is:

\[
M_{\text{SINR}^{(eq)}}(s) = 1 - (2\pi)^{-1/2} \sum_{k=0}^{m_{BG}+1} Q_k^{(1)}(s; \xi_1) \sum_{i=0}^{m_{BG}+1} Q_i^{(2)}(s; \xi_2) \sum_{j=0}^{m_{BG}+1} Q_{k,i,j}^{(3)}(s; \xi_3)
\]

where \( Q_k^{(1)}(\cdot; \cdot) \), \( Q_i^{(2)}(\cdot; \cdot) \), and \( Q_{k,i,j}^{(3)}(\cdot; \cdot) \) are defined in [4.83], and \( \xi_1 = m_{SD} / \bar{\gamma}_{SD} \), \( \xi_2 = m_{SR} / \bar{\gamma}_{SR} + m_{RD} / \bar{\gamma}_{RD} \), \( \xi_3 = m_{SD} / \bar{\gamma}_{SD} + m_{SR} / \bar{\gamma}_{SR} + m_{RD} / \bar{\gamma}_{RD} \).

From the MGF in (4.21), the diversity order can be obtained following the same line of sight as Proposition 4.2 and 4.3 by considering the homogeneous and heterogeneous scenarios.

**Proposition 4.6:** Let \( E_S = K_T E_T \), \( E_R = (1 - K_T) E_T \) and \( E_T = E_S + E_R \) with \( 0 < K_T < 1 \). Let \( E_t / N_0 = \tau_T (E_t / N_0) \). If \( E_t / N_0 \gg 1 \), the MGF of the equivalent SINR is approximated by (4.21) obtained by replacing \( Q_k^{(1)}(\cdot; \cdot) \), \( Q_i^{(2)}(\cdot; \cdot) \), \( Q_{k,i,j}^{(3)}(\cdot; \cdot) \) with \( Q_k^{(4)}(\cdot; \cdot) \), \( Q_i^{(5)}(\cdot; \cdot) \) and \( Q_{k,i,j}^{(6)}(\cdot; \cdot) \) defined in [4.83], respectively, as well as \( \bar{\gamma}_{XY} \) with \( \nu_{XY} \), where 
\[
\nu_{XY} = K_T \left( \kappa_{XY} / d_{XY}^{\nu_{XY}} \right)^2 \Omega_{XY} \text{ and } \nu_{RD} = (1 - K_T) \left( \kappa_{RD} / d_{RD}^{\nu_{RD}} \right)^2 \Omega_{RD}.
\]
Proposition 4.7: Let \( E_S = K_T E_T \), \( E_R = (1 - K_T) E_T \) and \( E_T = E_S + E_R \) with \( 0 < K_T < 1 \). Let the heterogeneous scenario, i.e., \( E_i / N_0 = \rho_i \) is kept constant. If \( E_T / N_0 \ll 1 \), the MGF in (4.21) is approximated by:

\[
M_{SINR^{(n)}}(s) \approx \left( \frac{E_T}{N_0} \right)^{-1/b_1} \left( \rho_1 \sigma_G^2 \right)^{1/b_1} \Gamma \left( \frac{1}{b_1} + 1 \right) s^{-1/b_1} \\
\times \left( \xi_1^{1/b_1} \sum_{k=0}^{m_{\text{en}}-1} \widehat{Q}_k^{(1)}(\xi_1) + \xi_2^{1/b_1} \sum_{i=0}^{m_{\text{en}}-1} \sum_{t=0}^{m_{\text{en}}-1} \widehat{Q}_{k,t}^{(2)}(\xi_2) - \xi_3^{1/b_1} \sum_{k=0}^{m_{\text{en}}-1} \sum_{i=0}^{m_{\text{en}}-1} \sum_{t=0}^{m_{\text{en}}-1} \widehat{Q}_{k,t}^{(3)}(\xi_3) \right)
\]

(4.22)

Proposition 4.8: The diversity order in a quasi-static interference scenario is equal to 0 under the assumptions of Proposition 4.6 and to \( 1/b_1 \) under the assumptions of Proposition 4.7.

**Fast-Varying Interference Scenario**

Consider the fast-varying scenario, where \( i_{t_z} = \sqrt{E_i} \sqrt{B_z} G_z \) are i.i.d. for \( Z = 1,2 \).

Lemma 4.4: The end-to-end error probability of the SC demodulator in (4.20) in quasi-static and fast-varying interference scenarios is the same.

Based on Lemma 4.4, the error probability of SC in the fast-varying scenario can be studied from Propositions 4.5-4.8. Also, the same performance in both interference scenarios is expected.

**4.1.8 Numerical and Simulation Results**

In this section, we show some numerical results to verify the accuracy of the proposed mathematical methodology against Monte Carlo simulations and to validate our findings. As an example, the following setup is considered. Dual-hop network): i) \( d_{SD} = 500 \text{m} \), \( d_{SR} = d_{RD} = 250 \text{m} \); ii) \( \kappa_{SD} = \kappa_{SR} = \kappa_{RD} = 1 \); iii) \( b_{SD} = 3 , b_{SR} = 1.5 , b_{RD} = 2 \); iv) \( \Omega_{SD} = \Omega_{SR} = \Omega_{RD} = 1 \); v) \( m_{SD} = m_{SR} = m_{RD} = 2 \); vii) MPSK modulation with \( M = 16 \); and vii) \( E_S = E_R = E_T / 2 \). Interfering network): i) \( \kappa_i = 1 \); ii) \( b_i = 3.5 \); iii) \( m_i = 2 \); iv) MPSK modulation with \( M = 16 \); v) synchronous transmission[4.47]; and vi) \( \lambda = \{ 10^{-13}, 10^{-8}, 10^{-7}, 10^{-6}, 10^{-5} \} \) interferers /\( m^2 \). As for the AWGN, \( N_0 = k_b T_0 \) where
\( k_b = 1.380650424^{-23} \) Joule/Kelvin is the Boltzmann's constant and \( T_0 = 290 \) Kelvin is the noise temperature. As for \( E_f \), two scenarios are considered: i) \( E_f = E_f^T \), which corresponds to the homogeneous case of, e.g., Proposition 4.2; and ii) \( E_f / N_0 = 190 \) dB, which corresponds to the heterogeneous case of, e.g., Proposition 4.3.

For assessing the accuracy of the mathematical analysis, the error probability is computed in four ways. 1) Monte Carlo Simulations. These are obtained by simulating the whole communication system, including modulator, channel and demodulator, without any a priori assumptions about the distribution of the network interference. The following procedure is used[4.47]: i) a finite circular area of (normalized) radius \( R_A \) around the origin, i.e., where \( D \) is located, is considered. The radius is chosen such that \( \lambda \pi R_A^2 = 100 \), in order to minimize the error committed in simulating an infinite bi-dimensional plane; ii) the number of interferers is generated according to a Poisson distribution with density \( \lambda \) and area \( \pi R_A^2 \); iii) the locations of the interferers are distributed following a uniform distribution over the circular region of area \( \pi R_A^2 \); iv) independent channel gains are generated for each interferer; and vi) the exact diversity demodulators are used at \( D \); 2) Semi-Analytical Framework. Consider the MRC demodulator of Section 4.3 as an example. These results are obtained by inserting the SINR of (4.10) in (4.5) and by numerically computing the expectations with respect to \( \mathbf{h} \) and \( \mathbf{B} \). The RVs \( \mathbf{B} \) are generated with the aid of [4.82]; 3) Analytical Framework. These results are obtained by inserting the MGF of the equivalent SINR of, e.g., (4.13) in (4.5) and by numerically computing the integral; and 4) Asymptotic Framework. These results are obtained by inserting the MGF of the asymptotic equivalent SINR of, e.g., Propositions 4.2, 4.3 in (4.5) and by numerically computing the integral. As for semi-analytical, analytical and asymptotic frameworks, \( \sigma_0^2 \) is computed using (4.2).

Selected numerical examples illustrating the end-to-end Average Symbol Error Probability (ASEP) at \( D \) are shown in Figs. 4.2-4.5. For ease of illustration, the results obtained with Monte Carlo Simulations are reported only for \( \lambda = \{10^{-13}, 10^{-7}, 10^{-5}\} \), while the results obtained with the Semi-Analytical Framework are reported only for \( \lambda = \{10^{-8}, 10^{-6}\} \). As for \( \lambda = 10^{-13} \), it is important to remark that it corresponds to a very sparse interfering network, which nearly corresponds to an interference-free scenario in the analyzed range of SNRs. This setup is studied to substantiate the frameworks for all possible values of \( \lambda \) and to show
the achievable lower-bound without interference in the SNR range of interest. In Figs. 4.5.2-4.5.5, in particular, the solid lines show the Analytical Framework, the dashed lines show the Asymptotic Framework and the markers show either Monte Carlo Simulations or the Semi-Analytical Framework depending on $\lambda$. Overall, we observe a good agreement between Monte Carlo simulations and the proposed mathematical frameworks. The numerical examples confirm the findings about the achievable diversity, as a function of the demodulator used at the destination and of the interference scenario.

![Graph showing the achieved lower-bound without interference in the SNR range of interest.](image)

**Fig 4.2** Interference–oblivious MRC demodulator with quasi–static interference in (a) homogeneous, (b) heterogeneous scenario

![Graph showing the achieved lower-bound with interference in the SNR range of interest.](image)

**Fig 4.3** Interference–oblivious MRC demodulator with fast–varying interference in (a) homogeneous, (b) heterogeneous scenario
4.2 Performance evaluation of cellular networks with randomly distributed relays

4.2.1 Introduction
The deployment of relays, as infrastructures without a wired backhaul connection, have been considered by IEEE 802.16j working group [4.2] and the Third Generation Partnership Project’s Long Term Evolution-Advanced (3GPP LTE-A) [4.3] to enhance the throughput and
coverage as a cost-effective solution in future cellular networks. Currently, practical systems usually consider half-duplex relays (the relays can either receive or transmit but not at the same time) to relay the message from the base stations (BSs) and the mobile terminals (MTs) using amplify-and-forward, decode-and-forward or demodulate-and-forward protocols [4.1], [4.51], [4.83], [4.84]. However, the relays deployed in the commercial multihop wireless network are not designed to specifically mitigate interference. Consequently, interference is one of the main performance limiting factors in the relay-aided wireless networks.

In this context, the performance of the relay-aided transmission in interference limited schemes have been an active field of research in the last few years. For example, in [4.51], the outage probability of the dual hop transmission is computed in the presence of randomly distributed interfering nodes around the destination. In Section 4.1-4.5, the achievable diversity order of the dual-hop cooperative relays are studied by assuming the interference follows symmetric alpha stable distribution at the destination and the performance of different receivers in interference-limited cooperative wireless networks are compared.

Even though the above cases studied give a clear understanding of the performance of relay-aided wireless networks in the presence of interference, all these studies assume that either the position of the BS or the position of the relay is fixed. Furthermore, the extra interference generated by the active relays are not considered in the system model of the cited papers, which is one of the main challenges faced by the relay deployment in the cellular networks [4.2].

Motivated by these considerations, in the present paper we use stochastic geometry to model the positions of the BSs, the MTs, and the relays to study the coverage probability of the relay-aided downlink transmission by considering the additive noise as well as the interferences generated by the BSs and the relays. In particular, the Poisson Point Process (PPP)-based approach [2.1] has been used, which is considered as accurate as other abstraction models [2.2] and leads to a tractable mathematical analysis for further study.

More precisely, both cooperative and non-cooperative transmission are considered. The MT can choose a BS or a relay as its serving node according to the association policy, while the serving nodes can serve multiple users using orthogonal resource blocks, e.g., time-frequency resource block in LTE [2.52]. Thus, the intra-cell interference is not considered in this paper. All the interferences are generated from the other-cell BSs or other relays using the same resource block as the serving node. Further, we assume all the BSs and relays operate in open access model but not with full load, i.e., all the MTs can connect to them
without any restrictions. Particularly, three association policies are studied. The coverage probabilities of the relay-aided wireless network with different association policies are investigated through extensive Monte Carlo simulations. The numerical results also show the benefit or loss caused by the deployment of relays compared to the non-cooperative cellular network.

4.2.2 System Model

As shown in Fig. 4.6.1, we consider a bi-dimensional network consisting of BSs and relays spatially distributed according to two independent PPPs $\Phi_{BS}$ and $\Phi_{R}$ with density $\lambda_{BS}$ and $\lambda_{R}$, respectively. The locations of the MTs are also modeled an an independent PPP $\Phi_{MT}$ with density $\lambda_{MT}$.

![Fig. 4.6.1 A relay-aided cellular network with PPP distributed BSs, relays, MTs, and cell boundary of BSs (line).](image)

Each MT can be served either by the BS or by the relay according to the association policies introduced in Section 4.6.3. Let a typical MT be denoted as $MT_0$. If $MT_0$ is associated to a BS, denoted as $BS_0$, then $BS_0$ would allocate a free resource block to serve $MT_0$ directly. In this case, the interfering signals are generated from other the BSs using the same resource block as $BS_0$. On the other hand, if $MT_0$ is tagged to a relay, denoted as $R_0$, the transmission is divided into two orthogonal time-slots [4.83], [4.84]. In the first time-slot, $R_0$
receives useful signals from its serving BS \( B_0^{(R)} \) occupying a free resource block of \( B_0^{(R)} \). The interferences received by \( R_0 \) are from other BSs using the same resource block as \( B_0^{(R)} \). In the second time-slot, \( R_0 \) uses the decode-and-forward protocol for relaying the detected message to \( M_0 \) using a free resource block of \( R_0 \). The interfering nodes in the second time-slot are other active relays using the same resource blocks as \( R_0 \). For simplicity, we assume that the relays are always served by the nearest BSs of the relays. It is worth mentioning that \( B_0 \) and \( B_0^{(R)} \) may not be the same BS.

**SINR**

If \( M_0 \) is served by \( B_0 \) directly using resource block \( m_0 \), the SINR at \( M_0 \) is:

\[
\text{SINR}_{M_0}^{(1)} = \frac{E_T h_{0T_0}^{-\alpha_{BS,D}}}{N_0 + i_{BS,D}}
\]

where i) \( E_T \) is the transmit power of \( B_0 \); ii) \( h_{0T_0}^{-\alpha_{BS,D}} \) is the channel propagation coefficient of the \( B_0 \)-to- \( M_0 \) link with path loss exponent \( \alpha_{BS,D} > 2 \), distance \( r_0 \) and the standard Rayleigh fading coefficient \( h_0 \sim \exp(1) \) [2.24]. For simplicity, independent and identically distributed (i.i.d.) Rayleigh fading channels are assumed throughout this paper; iii) \( N_0 \) is the variance of the Additive White Gaussian Noise (AWGN) at \( M_0 \); iv) \( i_{BS,D} \) is the cumulative interference at \( M_0 \), which can be formulated as follows:

\[
i_{BS,D} = \sum_{i \in \Phi_{BS}^{(1)} \setminus B_0} E_T h_i r_i^{-\alpha_{BS,D}} + \sum_{j \in \Phi_{BS}^{(2)}} E_j h_j r_j^{-\alpha_{BS,D}}
\]

where \( \Phi_{BS}^{(1)} \subseteq \Phi_{BS} \) represents the set of active BSs serving MTs directly using resource block \( m_0 \). All the BSs in \( \Phi_{BS}^{(1)} \) have transmit power \( E_T \); \( \Phi_{BS}^{(2)} \subseteq \Phi_{BS} \) denotes the set of BSs serving relays with resource block \( m_0 \); \( E_B = K_T E_T \), \( 0 < K_T < 1 \), is the transmit power of the BSs in \( \Phi_{BS}^{(2)} \).

When \( M_0 \) is served by \( R_0 \) using the dual hop link, \( R_0 \) receives the message from \( B_0^{(R)} \) in the first time slot, and the SINR of \( R_0 \) can be expressed as:
\[ \text{SINR}_{R_0} = \frac{E_{BS} \hat{h}_0 \hat{r}_0^{-\alpha_{BS,R}}}{N_0 + i_{BS,R}} \]  
\[ (4.6.3) \]

where i) \( \hat{h}_0 \hat{r}_0^{-\alpha_{BS,R}} \) is the channel gain of the BS\(_0\)-to-\( R_0 \) link with Rayleigh fading envelop \( \hat{h}_0 \), distance \( \hat{r}_0 \) and path loss exponent \( \alpha_{BS,R} > 2 \); ii) \( i_{BS,R} \) is the interference at \( R_0 \) similar to \( i_{BS,D} \) in (4.6.2):

\[ i_{BS,R} = \sum_{j \in \Phi_{BS}^{(2)}} E_j \hat{h}_j \hat{r}_j^{-\alpha_{BS,R}} + \sum_{j \in \Phi_{BS}^{(2)} \setminus \{BS(R_0)\}} E_{BS} \hat{h}_j \hat{r}_j^{-\alpha_{BS,R}} \]  
\[ (4.6.4) \]

In the second time-slot, \( MT_0 \) receives the relaying message from \( R_0 \), and the SINR in this case is:

\[ \text{SINR}_{MT_0}^{(2)} = \frac{E_{R} \hat{h}_0 \hat{r}_0^{-\alpha_R}}{N_0 + i_R} \]  
\[ (4.6.5) \]

where i) \( E_R = (1 - K_T) E_T \) is the transmit power of the relays; ii) \( \hat{h}_0 \hat{r}_0^{-\alpha_R} \) is the channel gain of the \( R_0 \)-to-\( MT_0 \) link with fading envelop \( \hat{h}_0 \), distance \( \hat{r}_0 \) and the path loss exponent \( \alpha_R > 2 \); iii) \( i_R \) is the aggregate interference at \( MT_0 \) which can be expressed as:

\[ i_R = \sum_{k \in \Phi_{R}^{(2)}} E_k \hat{h}_k \hat{r}_k^{-\alpha_R} \]  
\[ (4.6.6) \]

where \( \Phi_{R}^{(2)} \subseteq \Phi_{R} \) represents the set of active relays using the same resource block as \( R_0 \). It is worth mentioning that the transmission from the relay to the MT is not affected by interferences from BSs because of the orthogonal two time-slots transmission.

**Coverage Probability**

The BSs and relays are assumed to have \( M_{BS} \) and \( M_{R} \) orthogonal resource blocks which are chosen with equal probability. Since the MTs and the BSs are distributed according to two independent PPPs, the average number of MTs located in each cell is \( \lambda_{MT} / \lambda_{BS} [2.6] \). In order to avoid spatial blocking [4.85], i.e., the MTs cannot be served by the associated BSs because all the resource blocks of the serving BSs have been occupied, we assume \( \lambda_{MT} / \lambda_{BS} << M_{BS} \). Thus, the probability of spatial blocking when the MTs are tagged to the BSs is very small and can be neglected. Similarly, we assume \( \lambda_{MT} / \lambda_{R} << M_{R} \) to avoid the spatial blocking of the relays. With these assumptions, all the MTs can be served by their serving nodes and the BSs and relays are not operated with full load.
Using the association policies in Section 4.6.3, all the MTs are served by a BS or a relay, and the coverage probability of the system can be defined as:

\[ P_c = E_{\text{MT}, c |_{\text{MT}}} \left[ \mathbf{1}(\text{SINR}_{\text{MT}} > \beta) \right] \]  

(4.6.7)

where \( \mathbf{1}(\text{SINR}_{\text{MT}} > \beta) \) is defined in (4.6.8); \( \beta \) is the target SINR threshold. It is noticeable that if a MT is tagged to the dual hop link, it is in coverage when the SINRs of both links are above the threshold. In other words, the performance of the dual hop link depends on the weakest hop [4.8].

\[ \mathbf{1}(\text{SINR}_{\text{MT}} > \beta) = \begin{cases} 
\mathbf{1}(\text{SINR}_{\text{MT}}^{(1)} > \beta) & \text{MT} t \text{ is served by BS} \\
\mathbf{1}(\text{SINR}_{\text{R}} > \beta) \mathbf{1}(\text{SINR}_{\text{MT}}^{(2)} > \beta) & \text{MT} t \text{ is served by relay}
\end{cases} \]  

(4.6.8)

### 4.2.3 Association Policies

**Association 1**

Similar to Section 4.6.2, let \( BS_0 \) and \( R_0 \) denote the closest BS and closest relay to the MT ( \( MT_0 \) ), respectively. Let \( r_0 \) be the distance between \( BS_0 \) and \( MT_0 \). Let \( \bar{r}_0 \) be the distance from \( R_0 \) to \( MT_0 \). Then, using association 1, \( MT_0 \) is tagged to the node \( t \) defined as follows:

\[ t = \begin{cases} 
BS_0 & \text{if } r_0 \leq \bar{r}_0 \\
R_0 & \text{if } r_0 > \bar{r}_0
\end{cases} \]  

(4.6.9)

In other words, \( MT_0 \) is served by the closest serving node in the network. With a low implementation cost, the nearest distance association is widely considered in relay-aided wireless networks and multi-tier cellular networks.

**Association 2**

Let the same notations as Association 1. According to association 2, \( MT_0 \) is tagged to the node \( t \) defined as follows:

\[ t = \begin{cases} 
BS_0 & \text{if } B_{BS} E_{T} r_0^{-\alpha_{BS,V}} \geq B_{R} E_{T} \bar{r}_0^{-\alpha_{R}} \\
R_0 & \text{if } B_{BS} E_{T} r_0^{-\alpha_{BS,V}} < B_{R} E_{T} \bar{r}_0^{-\alpha_{R}}
\end{cases} \]  

(4.6.10)

where \( B_{BS} > 0 \) and \( B_{R} > 0 \) are the biasing factors of the BSs and relays. The MTs using Association 2 chooses the serving nodes based on the biased average received power by taking into account the channel propagation as well as the transmit power of the serving nodes. This association finds its rationale from the long-term averaged maximum biased-received-power association policy used for multi-tier cellular networks in [2.3].
**Association 3**

Using the same notations as in Association 1, and let $\hat{r}_0$ represent the distance from $B_{S_0}^{(R_0)}$ to $R_0$, where $B_{S_0}^{(R_0)}$ is the closest BS of $R_0$. From Association 3, $MT_0$ is tagged to the node $t$ defined as follows:

$$
t = \begin{cases} 
BS_0 & \text{if } \frac{B_{BS}E_T}{\alpha_{BS,p}} \geq B_R \cdot \min \left\{ \frac{E_{BS}}{\alpha_{BS,x}}, \frac{E_R}{\alpha_{R}} \right\} \\
R_0 & \text{if } \frac{B_{BS}E_T}{\alpha_{BS,p}} < B_R \cdot \min \left\{ \frac{E_{BS}}{\alpha_{BS,x}}, \frac{E_R}{\alpha_{R}} \right\}
\end{cases}
$$

(4.6.11)

Since the performance of the dual hop link depends on the weakest link, Association 3 compares the overall averaged received power of the direct link and the dual hop link at the cost of extra complexity.

Compared to Association 1, Associations 2 and 3 provide more flexibility by introducing the biasing factors. It is apparent that when $B_{BS} = 0$, the MTs will always choose the relay-aided transmission. When $B_R = 0$, the system reduces to the non-cooperative cellular networks.

These three association policies are based on the concept of increasing the average received power with a different implementation cost. However, choosing the link with a strong useful received power may also increase the interference received by other MTs in the network. Indeed, interference is one of the main limiting factors in future cellular networks [2.1]. In the next section, Monte Carlo simulations are used to study the coverage probability of the relay-aided wireless network using these three association policies.

**4.2.4 Simulation Results**

In this section, selected numerical examples are displayed to show the coverage probability of the relay-aided cellular network as described in Section 4.6.2. More specifically, as far as Monte Carlo simulations are concerned, the following seven-step methodology has been used:

- Step 1: A real bi-dimensional network in a finite circular area with radius $R$ is considered. The radius is chosen such that $\lambda_{BS} \pi R^2 = 3000$, i.e. around 3000 BSs are simulated in this area, in order to minimize the error committed by the truncation problem.
• Step 2: The BSs, relays, and MTs are generated in the simulated area according to three independent PPPs with density $\lambda_{BS}$, $\lambda_{R}$ and $\lambda_{MT}$, respectively.

• Step 3: For each MT, the association policies described in Section 4.6.3 are applied to determine the links and the resource blocks used to serve the MT, i.e., the transmission status of all BSs and relays are identified.

• Step 4: Independent channel gains are generated for all useful and interfering links between BSs and MTs, BSs and relays, and relays and MTs.

• Step 5: The SINRs of each MT and each active relay are computed as shown in (4.6.1), (4.6.3) and (4.6.5).

• Step 6: The coverage rate of this particular network deployment is obtained according to (4.6.7) and the SINRs computed in Step 5.

• Step 7: Finally, the coverage probability of the system is computed by repeating Step 2 - Step 6 for at least $10^n$ times.

For comparison, the coverage probability of a non-cooperative cellular network is also presented to show the benefit or loss of the deployment of relays. The MTs and the BSs are distributed in the simulated area according to two independent PPPs with the same density as in Step 2. The MTs in the non-cooperative cellular network are associated to their nearest BSs. All the BSs are transmitting with full power $E_T$.

In Fig. 4.6.2-4.6.5, we illustrate the coverage rate of the relay-aided architecture with a set of values of path loss exponents $\alpha_{BS,D}$, $\alpha_{BS,R}$, $\alpha_R$ and with a wide range of density of relays $\lambda_R$. In general, we consider the density of MTs $\lambda_{MT} = 10^{-4}$ while the density of BSs $\lambda_{BS} = 2 \times 10^{-5}$ and the number of resource blocks per BS $M_{BS} = 20$ throughout the simulations to avoid the spatial blocking described in Section 4.6.2. Since the study focuses on the interference limited network, we assume the total power $E_T / N_0 = 300$ dB. In addition, as far as the biasing factors in Association 2 and Association 3 are concerned, we consider $B_R = 1$ to be fixed and only $B_{BS}$ is changing.
Fig. 4.6.2 Comparison of the coverage probabilities as a function of the threshold $\beta$ with different association policies. Setup: $\lambda_R = 10^{-4}$, $M_R = 20$, $\alpha_{BS,D} = 3$, $\alpha_{BS,R} = 6$, $\alpha_r = 6$, $K_I = 0.5$.

In particular, Fig. 4.6.2 provides the coverage probability of three association policies when $\lambda_R = 5\lambda_{BS}$, i.e., a dense relay deployment compared to the BSs. It is shown that Association 1 with the lowest implementation cost gives the best performance, in terms of the coverage rate, among three association policies. On the other hand, when $B_{BS} = B_r$, using Association 2 and 3 almost provides the same performance as the cellular network without relays. The coverage rate increases, in general, for smaller biasing factor $B_{BS}$. In other words, serving more MTs via a dual hop link in the network, which decreases the interference, will improve the overall performance, though the average received signal from the dual hop link may be weaker than the direct link. This agrees with the observation that the path loss exponents in the dual hop link are much higher than that in the direct link. So the interfering nodes that use cooperative transmission and are far from the probe MT can be neglected.
Fig 4.6.3 Comparison of the coverage probabilities as a function of the threshold $\beta$ with different association policies. Setup: $\lambda_R = 2 \times 10^{-5}$, $M_R = 20$

, $a_{BS,D} = 3$, $a_{BS,R} = 6$, $a_R = 6$, $K_T = 0.5$

In Fig. 4.6.3, the coverage probabilities when the BSs and relays are distributed with the same density, i.e., $\lambda_R = \lambda_{BS}$, are displayed. In this network, changing the biasing factors, Association 2 and Association 3 may outperform Association 1. Furthermore, compared to Fig 4.6.2, the performance of Association 1 gets worse with the decreasing of $\lambda_R$.

Fig. 4.6.4 provides the coverage probability when $\lambda_R < \lambda_{BS}$, i.e., the relays are distributed sparer than the BSs. We notice that the performance of applying Association 1 drops dramatically compared to the results in Fig. 4.6.2, 4.6.3. Also in this case, the performance of using Association 3 remains as in denser relay deployment networks by carefully choosing the biasing factors.
Fig. 4.6.4 Comparison of the coverage probabilities as a function of the threshold $\beta$ with different association policies. Setup: $\lambda_R = 4 \times 10^{-6}$, $M_R = 100$, $\alpha_{BS,D} = 3$, $\alpha_{BS,R} = 6$, $\alpha_R = 6$, $K_T = 0.5$.

Fig. 4.6.5 Comparison of the coverage probabilities as a function of the threshold $\beta$ with different association policies. Setup: $\lambda_R = 10^{-4}$, $M_R = 20$, (a): $\alpha_{BS,D} = 3$, $\alpha_{BS,R} = 3$, $\alpha_R = 3$, (b): $\alpha_{BS,D} = 5$, $\alpha_{BS,R} = 3$, $\alpha_R = 3$
Finally, the impact of the path loss exponents is studied in Fig. 4.6.5. We observe that the benefit of the deployment of relays is limited when the path loss exponent of the direct link increases compared to the path loss exponents in dual hop links. Specifically, the cooperative transmission provides worse coverage probability compared to the non-cooperative cellular network in the interference limited case if $\alpha_{BS,D} > \alpha_{BS,R},\alpha_R$. This observation is interesting because it shows that many conclusions about the relay-aided architecture in noise limited environment are not applicable to the network with randomly distributed interferers.

From Monte Carlo simulations, it is observed that the performance of relay-aided transmission depends heavily on the path loss exponents of the channels, the density of the relays, as well as the association policies and the biasing factors. However, the simulation cannot provide insightful information on the system design and on the dependency of the system parameters to optimize. Thus, providing a mathematically tractable analysis on the performance of the relay-aided cellular network is of great value, which is our future research interest.
5. Decoupled Uplink and Downlink Access in Future Cellular Networks (DUDe)

5.1 Introduction

In order to keep up with the ever increasing network traffic, cellular networks are shifting from a single-tier homogeneous network approach to multi-tier heterogeneous networks (HetNets). HetNets, composed of different types of small cells (micro, pico and femto) and macro cells, have been a popular approach in the past few years as an efficient and scalable way to improve the network capacity in hotspots. However, most network technologies such as 3G or 4G were designed with Macro cells in mind and heterogeneity was just an afterthought. This dramatic change in cellular networks requires a fresh look on how present networks are deployed and what fundamental changes and improvements need to be done for future networks to operate efficiently.

Cellular networks have often been designed based on the downlink (DL); this is due to the fact that network traffic is mostly asymmetric in a way that the throughput required in the downlink is higher than the one required in the uplink. However, uplink is becoming more and more important with the growth of sensor networks and machine type communications (MTC) where the traffic is often uplink centric and also the increasing popularity of symmetric traffic applications, such as social networking, video calls, real-time video gaming, etc. As a consequence, the optimization of the uplink has become increasingly important and the question that we try to tackle in[5.1] is what improvements are possible to optimize the uplink of a highly densified HetNet?

5.2 Motivation

Cell association in cellular networks is normally based on the downlink received signal power only [5.2]. Despite differing UL and DL transmission powers and interference levels, this approach was sufficient in a homogeneous network where all the base stations (BS) are transmitting with the same or similar average power level. However, in HetNets where we have a large disparity in the transmit power of the different layers this approach is highly inefficient in terms of the uplink.

To understand this assertion we consider a typical HetNet scenario with a macro cell (Mcell) and a small cell (Scell), where in this paper we consider outdoor Scells. The DL coverage of the Mcell is much larger than the Scell due to the large difference in the transmit powers of both. However, in the UL all the transmitters, which are battery powered mobile devices, have
about the same transmit power and thus the same range. Therefore, a user equipment (UE) that is connected to a Mcell in the DL from which it receives the highest signal level might want to connect to a Scell in the UL where the pathloss is lower to that cell.

As HetNets become denser and small cells smaller, the transmit power disparity between macro and small cells is increasing and, as a consequence, the gap between the optimal DL and UL cell boundaries increases. For the sake of optimal network operation, this necessitates a new design approach which is the Downlink and Uplink Decoupling (DUDe) where the UL and DL are basically treated as separate network entities and a UE can connect to different serving nodes in the UL and DL.

The concept of DUDe has been discussed as a major component in future cellular networks in [5.3]-[5.5]. In [5.5], in particular, DUDe is considered as a part of a broader “device-centric” architectural vision, where the set of network nodes providing connectivity to a given device and the functions of these nodes in a particular communication session are tailored to that specific device and session. A study in [5.6] tackles the problem from an energy efficiency perspective where the UL/DL decoupling allows for more flexibility in switching-off some BSs and also for saving energy at the terminal side. In [5.7] Multi-Radio HetNets are discussed where all radio access technologies (RAT) like WiFi and LTE are managed under a single network and this can be considered as an extension to DUDe in future work where UL and DL can be scheduled on different RATs.

One technique that brings some fairness to the UL is “Range Extension” (RE) where the idea is to add a cell selection offset to the reference signals of the Scells to increase their coverage in order to offload some traffic from the Mcells [5.8]. However, using offsets greater than 3-6 dB may lead to high interference levels in the DL which is why techniques – like enhanced Inter-Cell Interference Coordination (eICIC) – have been developed to try to combat this type of interference [5.9]. Nevertheless, the RE technique is limited to moderate offset values due to the harsh interference in the DL. So DUDe would bring in the benefits of having very high RE offsets in the UL without the interference effects in the DL.

The main contribution of this work is to study the gains that can be achieved by the DUDe technique in terms of UL capacity and throughput and also to study the effects that this approach has on interference. We use a realistic scenario of a cellular network based on real-world planning/optimisation tools which, we believe, adds a lot of value and credibility to this work. In our best knowledge, this is the first work that assesses the benefits of decoupling UL and DL in a real world deployment.
5.3 Toy example showing the concept

In this study we drop the traditional UL/DL cell association based on DL received power (RP). We assume that while the downlink association is still based on DL RP, the uplink association is in fact based on pathloss. This apparently simple assumption in reality leads to radical changes in system design and architecture. One issue with this approach is when a UE has a link in 1 direction to a node (UL or DL) it needs a mechanism to allow the Acknowledgment process, channel estimation, etc. This would require major design changes. Therein we aim at studying whether the gains of DUDe justify such major changes.

DUDe results in different cell boundaries in the UL and DL in a HetNet scenario where a UE in the region between the UL and DL cell boundaries will be connected to the Scell and Mcell in the UL and DL respectively as shown in Figure 5.1. We will focus on the gains in the UL as this is the main motive for applying this technique. Note that DL capacities are not affected since the association remains unchanged.

In this section, we consider a two cell network model composed of a Mcell and a Scell to present the advantages of DUDe in a simplified way. The model is used to study two cases; the first case is a noise limited scenario with only one UE, to show the benefits in terms of uplink UE capacity. The second case is an interference limited scenario where there are three UEs in the network to show the benefits in reducing the interference. The two cases are explained in details below.

![System model for UL/DL decoupling.](image)

**Case 1 (noise limited)**

In this case we have one UE moving from the Scell vicinity towards the Mcell and the UE UL rate is calculated for two cases; the first is the conventional case where cell selection is based on the DL received power so the UE performs a Handover (UL & DL) from the Scell to the
Mcell when passing the DL cell border (shown in Figure 5.1) and the second case is where the UL cell selection is based on the PL where the UE is still connected to the Scell until passing the UL cell border which represents the DUDe technique.

Neglecting, for simplicity, fading and shadowing and normalizing various quantities, the UL rate calculation is based on the below equations:

\[ R = BW \log_2(1 + SNR) \]

\[ SNR = \frac{P_{\text{ue}}}{N d^\alpha} \]  

(5.1)

Here, R is the rate; SNR is the signal to noise ratio, P_{\text{ue}} is the UE transmit power and N is the noise power which is considered to be 0 dBm. BW is the bandwidth and is considered to be unity for simplicity. The distance based PL is dependent on the distance d and the pathloss exponent \( \alpha \).

We now calculate the UL rate for a UE moving from the Scell towards the Mcell for the two cell association methods, assuming, P_{\text{ue}} to be 20 dBm and the Scell and Mcell to have a PL exponent of 3.6 and 4 respectively. Finally, the Mcell and Scell have a transmit power of 46 and 23 dBm respectively.

Figure 5.2 illustrates the UL normalized rate for the PL and RP cell association cases. And it shows that the PL case has a higher performance in the area between the DL cell border and the UL cell border since in that area the UE has a lower pathloss to the Scell, thus obtaining a higher rate when connected to the Scell. The two curves are the same outside that area since the PL and RP cell association result in selecting the same cell.

![Figure 5.2 UE rate comparison between the DL Received Power (RP) case and the Pathloss (PL) case.](image)
Case 2 (interference limited)

In this case, we have the same setup as the previous one but with three UEs instead of only one UE as shown in Figure 5.1. We calculate the overall UL rate of the network using the PL based cell association where UE2 is connected to the Scell in the UL and then using the RP based cell association where UE2 is connected to the Mcell in the UL.

UE1 is always connected to the Scell in the UL and UE3 is always connected to the Mcell in the UL.

\[ R = BW \log_2(1 + SIR) \]
\[ SNR = \frac{P_{ue}}{N d^\alpha} \]  

(5.2)

The UL rate is calculated based on the above equation, where SIR is the Signal to Interference Ratio (we neglect the noise for simplicity). The total normalized UL rate (RT) is the sum of the normalized UL rate at the Mcell (RM) and the Scell (RS) which means the UL rate of the whole system (RT = RM + RS).

We use the same parameters as case 1 and setting d1, d2, d3, and d4 in Figure 1 to 10, 25, 80, and 100 respectively. So calculating RT in the PL case yields RT = 0.46 + 0.54 = 1 and in the RP case RT = 0.34 + 0.33 = 0.67. We can see that RT is almost 50% higher in the PL case for the following reasons.

- UE2 in the PL case has a lower PL to the Scell which means that UE2 has a better channel to the Scell and in turn gets a better rate when connected to it.
- UE2 causes less interference to the Mcell in the PL case than the interference it causes to the Scell in the RP case for the same reason as above, so the interference level in the network is lower and in turn the rate is higher.

5.4 Evaluation of the basic concept of DUDe

In this section we present our realistic simulation setup which is based on an existing cellular network and we use this setup to validate our findings and illustrate the gains from the studied concept.

5.4.1 System model

In our simulations we use the Multi-technology radio planning tool Atoll [5.10] in conjunction with a high resolution 3D ray tracing pathloss prediction model [5.11]. The model takes into account clutter, terrain and building data. This guarantees a realistic and accurate propagation model.
Atoll has the capability of performing system level simulations where a simulation is a snapshot of the LTE network. For each simulation, it generates a user distribution using a Monte Carlo algorithm. The user distribution is based on traffic data extracted from the real network. Resource allocation in each simulation is carried out over a duration of 1 second (100 frames).

As deployment setup, we use a Vodafone LTE small cell test bed network that is up and running in the London area. The test network covers an area of approximately one square kilometer. We use this existing test bed to simulate a relatively dense HetNet scenario. The considered network is shown in Figure 3 where the black shapes are macro sites and the red circles are small cells which are considered to be pico cells. We consider a realistic user distribution based on traffic data from the field trial network in peak times. The distribution is up-scaled to simulate a high user density. We use an uplink power control algorithm where each cell has a predefined interference upper limit. If the UL received interference at a cell is higher than this limit the cell signals the neighboring cells to lower the UL transmit power of their UEs in order to lower the interference level at that cell.

We simulate two cases; the first case is where the UL cell association is based on PL which represents the DUDe technique. The other case is where the UL cell association is based on the DL Reference Signal Received Power (RSRP) which is the conventional LTE procedure [5.2]. In the DL RSRP case we simulate low and high power Scell cases to understand the gains of the PL approach compared to the DL RSRP approach with different Scell sizes. As pointed out before, all the results in the next section will focus on the UL performance. The simulation parameters are listed in Table 5.1 where we consider an LTE deployment.

One deployment issue is that a UE connected to different nodes in the UL and DL needs a way to send Acknowledgment, pilot and relevant control signaling to its DL node with which it has no UL established. A possible way is to route the data to the UL node and through the backhaul to the DL node and vice versa with receiving control signals from the UL node. We assume an ideal backhaul where control signals are delivered with no notable delay. Non-ideal backhaul operation and alternative control signaling delivery mechanisms are left for future work.
Figure 5.3 Vodafone’s LTE small cell test network in London.

Table 5.1 Simulation parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Operating frequency</td>
<td>2.6 GHz (co-channel deployment)</td>
</tr>
<tr>
<td>Bandwidth</td>
<td>20 MHz (100 frequency blocks)</td>
</tr>
<tr>
<td>Network deployment</td>
<td>5 Mcells and 64 Scells distributed in the test area as shown in Figure 5.3.</td>
</tr>
<tr>
<td>User distribution</td>
<td>560 UEs distributed according to traffic maps read from a live network</td>
</tr>
<tr>
<td>Scheduler</td>
<td>Proportional fair</td>
</tr>
<tr>
<td>Simulation time</td>
<td>50 simulation runs with 1 second each.</td>
</tr>
<tr>
<td>Traffic model</td>
<td>Full buffer</td>
</tr>
<tr>
<td>Propagation model</td>
<td>3D ray-tracing model</td>
</tr>
<tr>
<td>Max. transmit power</td>
<td>Macro=46 dBm, High power Pico = 30 dBm, Low power Pico = 20 dBm, UE= 20 dBm.</td>
</tr>
<tr>
<td>Antenna system</td>
<td>Macro: 2Tx, 2Rx, 17.8 dBi gain Pico: 2Tx, 2Rx, 4 dBi gain UE: 1Tx, 1Rx, 0 dBi gain</td>
</tr>
<tr>
<td>UE's mobility</td>
<td>Pedestrian (3km/h)</td>
</tr>
<tr>
<td>Supported UL modulation schemes</td>
<td>QPSK, 16 QAM, 64 QAM</td>
</tr>
</tbody>
</table>
5.4.2 Results

In this section, we present results comparing three cases:

- DL RP based cell association where Scells are Pico cells (Pcells) with low transmit power (LP) of 20 dBm. This case is referred to as DL-LP.
- DL RP based cell association where Scells are Pcells with high transmit power (HP) of 30 dBm. This case is referred to as DL-HP.
- Pathloss based cell association which represents the Downlink and Uplink Decoupling (DUDe) (Pico transmit power is irrelevant as cell association is not based on DL RP).

![Figure 5.4 Uplink coverage of the DL_LP (left), DL_HP (middle) and DUDe (right) cases where green and red represent the Macro and Pico cells coverage respectively.]

Figure 5.4 illustrates the UL coverage of the Pcell layer (red) and Mcell layer (green) for the above three cases; it shows a much larger coverage for the Pcells in the DUDe case which ensures a more homogeneous distribution of UEs between the nodes which, in turn, results in a much more efficient use of the resources as will be shown in the following results.

In our simulations we define a UE minimum and maximum throughput demand where basically a UE has to reach the minimum throughput requirement to be able to transmit its data otherwise it is considered in outage. On the other hand the maximum throughput demand puts a limit to the amount of throughput that each UE can get, so setting a high value for it helps in simulating a highly loaded network.

The used scheduler tries first to satisfy the minimum throughput requirements for all the UEs and then distributes the remaining resources among the UEs to satisfy the maximum throughput demand of each UE according to the proportional fair criterion.
Figure 5.5 shows the effect of adding Pico cells on the 5th percentile UL throughput for the different cases. Pcells are all placed in their respective location as shown in Figure 5.3 but they are all switched off at the beginning and are activated one by one to understand the effect of increasing the number of Pcells in each case. In these results, we set the minimum throughput requirement to a relatively low value (100Kb/s) to show how the 5th percentile throughput evolves in the different cases without the constraint of a high minimum throughput requirement.

In the DUDe case, we see that the 5th percentile throughput is increasing with the number of Pcells. This is due to the fact that Pcells have a large coverage in the UL so they serve a large number of UEs and in turn have a big effect on the 5th percentile throughput. As the number of Pcells increases we notice that the 5th percentile UEs throughput starts to saturate as they are more limited by the channel quality and transmit power. So the extra capacity offered by adding more Pcells is used to serve the UEs with better channel conditions. Looking at the case of DL-LP and DL-HP, we see that adding Pcells has little effect on the 5th percentile throughput as Pcells have very limited coverage so their effect is more in the 90th percentile throughput rather than the 5th percentile.

Moreover; we see that the 5th percentile throughput is fluctuating as we increase the number of Pcells. This is basically due to the high interference that the Pcells UEs create to the Mcell cell edge UEs since these UEs are closer to the Pcells so they suffer from a high level of interference. We see this effect more clearly in the DL-LP case where the throughput starts to decrease after a certain point. On the contrary, in the DUDe case we see that the throughput is increasing more stably since the UEs always connect to the node to which they have the
lowest PL which guarantees a lower interference level as explained in Section II. In the next results all the Pcells in the test network are activated.

Table 5. 2 Average number of UEs per Node (Macro and Pico cells) for the three cases.

<table>
<thead>
<tr>
<th></th>
<th>DL-LP</th>
<th>DL-HP</th>
<th>DUDc</th>
</tr>
</thead>
<tbody>
<tr>
<td>Macro cell</td>
<td>81</td>
<td>50</td>
<td>13</td>
</tr>
<tr>
<td>Pico cell</td>
<td>2</td>
<td>4</td>
<td>8</td>
</tr>
</tbody>
</table>

Table 5. 2 shows the average number of UEs per cell where we calculate the average for the Mcells and Pcells separately having a constant total number of UEs (560) for all cases. The table shows how most of the UEs are connected to the Mcell in the DL-LP and DL-HP cases and the Pcells are under-utilized. On the other hand in the DUDc case the UEs are distributed in a more homogeneous way among the Mcells and Pcells which ensures much more efficient resource utilization.

![Figure 5.6](image)

Figure 5.6 5th, 50th and 90th percentile comparison of DL-LP, DL-HP and DUDc cases.

For the results in Figure 5.6 we set a minimum and maximum throughput demand of 200 Kb/s and 20 Mb/s respectively. The figure shows the 5th, 50th, and 90th percentile UE throughput for the three cases in comparison. The 5th percentile UL throughput in the DUDc case is increased by more than 200% compared to the DL-LP case and by 100% compared to the DL-HP. As for the 50th percentile UL throughput, the DUDc case has a gain of more than 600% compared to the DL-LP case and more than a 100% compared to the DL-HP case. The gains in the 5th and 50th percentile are resulting from the higher coverage of the Pcells in the DUDc case which results in a better distribution of the UEs among the nodes and a much more efficient usage of the resources. Also the fact that the UEs connect to the node to which
they have the lowest PL helps in reducing the UL interference as shown before. This results in a higher UE Signal to Noise and Interference Ratio (SINR) that allows the UEs to use a higher modulation scheme and in turn achieve a better utilization of the resources and a higher throughput.

Looking at the 90th percentile UL throughput we see that the DL-HP case achieves the highest throughput which can be explained by the fact that Pcells serve less UEs than the DUDe case then these UEs get a high throughput but on the expense of the 5th and 50th percentile UEs. Interestingly the DL-HP case achieves a higher 90th percentile throughput than the DL-LP case which seems counter intuitive. This can be explained by the fact that Pcells in the DL-LP case serve even less UEs than the Pcells in the DL-HP case so the effect of the Pcells in the DL-LP case is noticeable even after the 90th percentile. So if we look at the 98th percentile throughput in the DL-LP case it’s 15 Mb/s whereas in the DL-HP case it’s 10 Mb/s which shows that the effect of the Pcells in the DL-LP case is on a very limited number of UEs.

The gains of the 5th and 50th percentiles are comparable to the results shown in [5.12] where the authors apply a high Range Extension (RE) value to the Pcells and they get a two times gain in the 5th and 50th percentile throughput. The RE technique basically works in the same direction as decoupling the UL and DL in the sense that it results in an increased coverage in the UL. The disadvantage of RE is that the interference level in the DL increases aggressively as the RE bias increases which requires the usage of interference management techniques as mentioned before which is not required in the UL/DL decoupling since the UL and DL are treated as two different networks in this technique.

![Figure 5.7 Average outage rate of Macro and Small cell layers comparison between the DL-LP, DL-HP and DUDe cases.](attachment:image.png)
Figure 5.7 represents the average outage rate for the Mcell layer and the Pcell layer for the three cases or in other words the percentage of the UEs that fail to achieve the minimum throughput demand (1 Mb/s) out of the total number of connected UEs to a certain node. Since the simulated scenario is considered to be a highly dense scenario it requires a very efficient use of resources in order to satisfy the high requirements of the UEs. As seen in the figure the Macro layer has a very high outage rate (more than 90%) in the DL-HP and DL-LP cases which is basically explained by the fact that the Macro layer is very congested in the UL as seen in Table 2 so Mcells do not have enough resources to serve all their UEs with a high throughput level. However, in the DUDe case, UEs are distributed more evenly between the nodes so the outage rate in that case is low (less than 10%) for both Macro and Pico cell layers.

These results clearly show that decoupling UL and DL where UL is based on PL is a promising candidate for future networks where the network load is expected to increase in the UL and where providing a consistent and ubiquitous service to all UEs in different network deployments and UE densities is a priority. This technique would also allow freeing up spectrum resources in the UL which could be used for DL purposes.

So far in this section, we presented an assessment of the UL/DL decoupling concept (DUDe) in a dense HetNet deployment. We started by a simplified model to highlight the motives and gains of this concept and then we presented simulation results based on a live Vodafone LTE test network deployment in London. The simulations used a high resolution ray tracing propagation model and user distributions based on network measurements which make this model highly realistic and providing a much better view on the effects of deploying this technique in the real world than normal system level simulations. The gains are very high in a dense HetNet deployment where this technique can achieve between 100% and 200% improvement in the 5th percentile UL throughput and even more than that in the 50th percentile throughput. Also, we have shown that the outage rate is decreased dramatically in networks with high minimum throughput requirements where the outage rate is decreased from 90% to below 10% on the Macro layer. We believe that the DUDe technique is a strong candidate for 5G architecture designs and it can be very useful in many applications like Machine Type Communications (MTC) where Uplink optimization is very critical. The next section will include an evolution of the DUDe concept including cell load and backhaul awareness.
5.5 Load and backhaul aware decoupled access (DUDe 2.0)

5.5.1 Motivation

Driven by an increasing density of small cells in heterogeneous 4G systems, it was recently shown that the traditional strategy of handing over up (UL) and downlink (DL) simultaneously based on downlink received power is significantly capacity suboptimal. Indeed, the UL performance gains for cell-edge users due to decoupling the DL and UL cell association were consistently shown to be in the order of 200-300% [5.1].

These capacity improvements in the UL are very timely since the UL traffic has been growing over past years with an unprecedented rate. This trend is driven by new applications which generate symmetric traffic, such as real-time gaming and video calls. In addition, the emerging array of social networking applications as well as machine-to-machine technologies generates more UL traffic than DL in an uncorrelated fashion. The optimization of the UL, particularly for disadvantaged users at the cell edge, is thus of highest important to a consistent quality of experience in emerging 5G systems.

The decoupling is facilitated by the fact that the degree of heterogeneity has increased dramatically over past years and is expected to grow further as part of 4G and 5G rollouts. This shift from a single-tier homogeneous network towards multi-tier heterogeneous networks (HetNets) composed of different types of small cells (Micro, Pico and Femto) comes along with the unique opportunity to have ample connections available at any point and time. This, in turn, facilitates our purpose of decoupling the UL from the DL and the thereby achieved performance gains.

The concept is shown in Figure 5.1 where the Small Cell (Scell) has DL and UL cell borders which are defined by the DL received power and pathloss respectively; a UE between these two borders will tend to connect to the Scell in the UL and to the Macro cell (Mcell) in the DL as shown in the figure.

Some prior art is emerging in this field[5.1] and [5.13], but have so far assumed that the cell association strategy is based on the link quality in each direction. That is, the decision to handover the DL is (and has been) based solely on the DL received power; whereas the decision to handover the UL is based solely on the UL pathloss. The system assumptions were to some extent ideal in that neither the cell load nor the backhauling capabilities have been taken into account – both of which have an impact onto the actual performance gains under more realistic operating conditions. This shortcoming is addressed in this paper at
hand, where we proceed to outline prior related art as well as a summary of our technical contributions.

The concept of downlink/uplink decoupling (DUDe) has been discussed as a major component in future cellular networks in [5.3]-[5.5]. In [5.5], in particular, DUDe is considered as a part of a broader “device-centric” architectural vision, where the set of network nodes providing connectivity to a given device and the functions of these nodes in a particular communication session are tailored to that specific device and session. In [5.14] and [5.15], backhaul aware cell association was considered but only from a DL perspective. In [5.16] and [5.17], load aware cell association was studied in the DL as well. In [5.18], the authors study UL cell association in a game theoretic approach to optimize the packet success rate of the UEs.

In contrast to prior art, this paper focuses on the cell association algorithm where we argue that UL pathloss alone is not sufficient to efficiently apply DUDe. Notably, the association algorithm ought also to consider the overall load of the cell(s). Furthermore, since DUDe requires significant backhauling support, we also condition association with backhauling capacity. Therefore, instead of taking the decision based only on link quality, the system now considers the link quality, the cell load and the cell backhaul capacity. We then use a realistic scenario of a cellular network based on Vodafone’s real-world planning/optimisation tools which, we believe, adds a lot of value and credibility to this work. We give special attention to UL power control where we show that the performance depends greatly on the power control settings. We use a flow level traffic model that is more realistic than the full buffer model considered in prior art. The results are then discussed and evaluated in great details, thus offering unique insights into the performance trends of the emerging decoupling concept.

5.5.2 System model

We consider the UL of a HetNet where, as deployment setup, we use the Vodafone LTE small cell test bed network that is up and running in the London area. The test network covers an area of approximately one square kilometre. We use this existing test bed to simulate a relatively dense HetNet scenario.

The considered network is still shown in 3 where the black shapes are Macro sites and the red circles are Small cells where in total we have B cells. We consider a realistic user distribution based on traffic data from the field trial network in peak times where the total number of users is Nu. Network traffic is modelled on a flow level where flows represent individual file or data transfers e.g. video, audio or generic file uploads. This model reflects a much more realistic
traffic model than the full buffer model considered in [5.1]. We assume that a flow of size $\rho$ arrives to a UE’s queue after a certain period “wait time” $T_W$. $T_W$ and $\rho$ are exponentially distributed with certain mean values. UEs experience a different $T_W$ each time a flow transmission is finished. The radio link quality is determined by many factors including pathloss, fading, interference, and transmit power of the UEs. The UL SINR of UE $i$ connected to BS $j$ is given by:

$$\text{SINR}_{ij} = \frac{h_{ij}P_i}{N + I}$$  \hspace{1cm} (5.3)

where $P_i$ is the $i$th UE transmit power, $h_{ij}$ incorporates pathloss, shadowing and fast fading between UE $i$ and BS $j$, $N$ is the noise power and $I$ is the UL intercell interference. We characterize the achievable data rate using the Shannon formula where $BW$ represents the system bandwidth:

$$C_{ij}^{\text{Access}} = BW \log_2(1 + \text{SINR}_{ij}).$$  \hspace{1cm} (5.4)

Uplink power control for the UEs follows the 3GPP specifications [5.19], where we consider open loop power control which is given by:

$$P_{UE} = \min\{P_{\text{MAX}}, 10\log_{10}(M) + P_0 + \alpha.L\},$$  \hspace{1cm} (5.5)

where $P_{\text{MAX}}$ is the maximum permittable transmit power of the UE, $M$ is the number of physical resource blocks (PRBs) assigned to the UE, $P_0$ is a normalized power, $\alpha$ is the pathloss compensation factor and $L$ is the pathloss towards the serving cell.

However, the power control algorithm does not account for inter-cell interference which, as we will show in the results, affects greatly the UL performance. The effect is more pronounced when load balancing takes place since UEs connect to a suboptimal cell so they are more vulnerable to interference. Therefore we will use an interference aware power control algorithm which sets a limit to the transmit power of the UEs depending on the interference level that the UE causes to the closest neighboring cell. Similar algorithms have been proposed in the literature such as [5.21]. The algorithm is as follows:

$$P_{UE} = \min\{P_{\text{MAX}}, 10\log_{10}(M) + P_0 + \alpha.L + L_0 + L_s + 10\log_{10}(M)\}$$  \hspace{1cm} (5.6)

where $L_0$ represents the UL interference power spectral density (PSD) target for the UE and $L_s$ is the pathloss towards the most interfered cell by the UE. This allows us to control the interference level in the system by changing $L_0$.

In a real world deployment, the Scell backhaul is always an issue since outdoor Scells are usually mounted on street furniture where there is no guaranteed wired connection or line-of-sight to the Mcell. Furthermore with the increasing bit rates provided by access technologies the bottleneck is moving slowly from the access network to the backhaul. We consider that all
cells in the test network have a limited backhaul capacity $C_{jk}^{bk}$ where, naturally, Scells would have tighter backhaul constraint than Mcells.

5.5.3 Cell association algorithm

In our previous study [5.1] we have considered the UL cell association to be based on pathloss (PL) which showed very high performance improvements that were mainly due to the load balancing effect and the improved link quality of the UEs. We extend this approach to include the cell load and backhaul capacity in the decision criterion; consequently instead of taking the decision based only on link quality the UE considers the link quality, the cell load and the cell backhaul capacity. This approach makes sense since in real networks users are distributed in a non-uniform way where a UE that is close to a congested cell might be better off connecting to a cell that is further but less congested.

We consider a cell association criterion that was considered in [5.16] in the DL. We extend this by applying it to the UL and including the backhaul capacity so that the optimal BS chosen by UE $i$ is given by $s(i)$:

$$s(i) = \arg \max_{j \in B} \left( 1 - \eta_j \right) C_{ij}^{Max}$$

(5.7)

where $C_{ij}^{Max} = \min\{C_{ij}^{Access}, C_{ij}^{bk}\}$, $\eta_j$ is the $j$th BS load which is reflected in [5.16] as being the average resource utilization per cell. We found that this approach for $\eta_j$ works fine in the DL whereas in the UL the situation is different since the UEs are power limited which means that a UE with bad channel conditions would not be able to transmit on a large number of resource blocks. This would result in a low utilization of the resources of the cell even though this cell could be serving many UEs. Therefore the cell utilization is a poor metric to characterize the cell load in the UL and we resort to a different way of estimating the load. Notably, since the flow arrival is exponentially distributed and assuming the system to be stationary, the stationary distribution of the number of flows $N_j$ is identical to that of an M/GI/1 multi-class processor sharing system [5.20]. The average number of flows is then given by $[N_j] = \frac{\eta_j}{1 - \eta_j}$, which yields $\eta_j = \frac{E[N_j]}{E[N_j] + 1}$. Inserting $\eta_j$ into (5.5) yields:

$$s(i) = \arg \max_{j \in B} \frac{C_{ij}^{Max}}{E[N_j] + 1}$$

(5.8)

The cell association criterion in (5.8) will be used for the rest of this section. We consider a fully distributed algorithm where the main idea is that a UE does not need to stay connected to one BS in the UL all the time. Instead a UE can keep its anchor DL cell and every time the UE
has data (flow) to transmit in the UL, the UE connects to the cell with the highest criterion according to (5.8).

The algorithm thus functions as follows: The BSs broadcast their load $E[N_j]$ and backhaul capacity $C_{jk}$. All UEs in the system start with an exponentially distributed wait time ($T_w$) after which a UE has a flow of size $\rho$ to transmit. The UE uses the criterion in (5.6) to find the best cell to connect to and after finishing its transmission the UE disconnects from the cell and goes idle for another random period $T_w$; thereupon the operation is repeated. The steps are detailed in Algorithm 5.1.

**Algorithm 5.1: Load/backhaul aware UL cell association**

1. BSs broadcast $E[N_j]$ and $C_{bk}$ periodically.
2. UEs ($1\ldots N_u$) are idle for a random $T_w(1\ldots N_u)$.
3. for Number of subframes
4. for each idle UE
5. if $T_w = 0$
6. UE_queue = $\rho$.
7. UE connects to BS (i) according to (5.6)
8. UE is scheduled in BS (i) until UE_queue = 0.
9. UE goes idle for a random $T_W$.
10. else
11. $T_w = T_w - 1$
12. end for
13. end for

**5.5.4 Results**

In our simulations we use the deployment setup of the Vodafone LTE test network in the London area. The setup consists of 5 Mcells and 21 outdoor Scells. Our propagation model is based on a high resolution 3D ray tracing pathloss prediction model. The model takes into account clutter, terrain and building data. This guarantees a realistic and accurate propagation model. The user distribution is based on traffic data extracted from the real network.

We consider three power control settings:

- Loose power control with full pathloss compensation. We use (5.3) where $(\alpha, P_0)$ are set to (1, -80 dBm). This is referred to as **Setting 1**.
- Conservative power control with partial pathloss compensation. We use (5.3) where $(\alpha, P_0)$ are set to (0.6, -70 dBm). This is referred to as **Setting 2**.
- Interference aware power control where we use (5.4) and set $(\alpha, P_0)$ to $(1, -80 \text{ dBm})$
and $f_0$ to $-100 \text{ dBm}$.

Table 5.3 Simulation parameters

<table>
<thead>
<tr>
<th>Operating frequency</th>
<th>2.6 GHz (co-channel deployment)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bandwidth</td>
<td>20 MHz (100 frequency blocks)</td>
</tr>
<tr>
<td>Network deployment</td>
<td>5 Mcells and 21 Scells distributed in the test area.</td>
</tr>
<tr>
<td>User distribution</td>
<td>330 UEs distributed according to traffic maps read from a live network</td>
</tr>
<tr>
<td>Scheduler</td>
<td>Proportional fair</td>
</tr>
<tr>
<td>Simulation time</td>
<td>10 seconds (10,000 subframes)</td>
</tr>
<tr>
<td>Traffic model</td>
<td>Flow level traffic. Mean flow size = 1 Mbit. Mean wait time = 100 ms.</td>
</tr>
<tr>
<td>Propagation model</td>
<td>3D ray-tracing model</td>
</tr>
<tr>
<td>Max. transmit power</td>
<td>Mcell = 46 dBm, Scell power = 30dBm, UE = 20 dBm.</td>
</tr>
<tr>
<td>Antenna system</td>
<td>Macro: 2Tx, 2Rx, 17.8 dBi gain Pico: 2Tx, 2Rx, 4 dBi gain UE: 1Tx, 1Rx, 0 dBi gain</td>
</tr>
<tr>
<td>UEs mobility</td>
<td>Pedestrian (3km/h)</td>
</tr>
<tr>
<td>Supported UL modulation schemes</td>
<td>QPSK, 16 QAM, 64 QAM</td>
</tr>
</tbody>
</table>

We compare 3 UL cell association cases:
- Cell association based on the DL Reference Signal Received Power (RSRP) which is the conventional LTE procedure [5.19]. This case is termed **DL-RSRP**.
- Cell association based on the pathloss which represents the DUDe algorithm as considered in [5.1] and is termed **DUDe**.
- Cell association based on Algorithm 5.1 which considers the cell load and backhaul capacity on top of the conventional DUDe. This case is termed **DUDe-Load**.

As pointed out before, all the results in the next section will focus on the UL performance. The simulation parameters are listed in Table 5.3 where we consider an outdoor LTE deployment.
Initially we assume having an ideal backhaul (i.e. no limit on the backhaul capacity) on all the cells in order to study the load balancing effect. We start by comparing the throughput results with different power control settings according to Setting 1 and Setting 2.

![Figure 5.8 Throughput percentiles for the three cases with power control Setting 1 and 2.](image)

The throughput results for the three cases in comparison are shown in Figure 5.8. Comparing DUDe to DL-RSRP, we see similar gains as in [5.1] where the 5th and 50th percentiles are increased by more than 100% and 150% respectively for both power settings. The gains are due to the load balancing effect of DUDe and the better link quality as UEs connect to the cells to which they have the lowest PL. The 90th percentile throughput is less in DUDe than DL-RSRP as in the latter case only a few UEs are served by the Scells; therefore these UEs achieve a high throughput.

We notice also that DUDe-Load is more affected, in terms of 5th and 50th percentiles, by the change in the power settings than DUDe. This is due mainly to the fact that UEs connect to suboptimal cell in terms of PL due to the load balancing effect which makes these UEs more vulnerable to interference and more affected by the other UEs transmit power.

We then compare DUDe and DUDe-Load starting by Setting 1 where we see that the 5th percentile throughput is reduced by about 20% in the DUDe-Load case while the 50th percentile is increased by 40% compared to DUDe. The loose power control causes the interference level to increase which has a negative effect on the cell edge UEs as explained below.
This result shows that cell edge UEs (5th percentile) are better connected to a loaded cell to which they have the better link quality than connecting to an unloaded cell with a worse channel. On the other hand the 50th percentile UEs can afford a reduced channel quality and with the higher power headroom they actually achieve a high gain by using the extra resources provided by the load balancing effect of DUDe-Load. Finally, the figure also shows a loss of about 20% in the 90th percentile throughput which is logical since load balancing is always a trade-off between peak and (cell-edge/average) throughput.

Then we compare DUDe and DUDe-Load for Setting 2 where the 5th percentile throughput in DUDe-Load is improved by about 40% over DUDe whereas the 50th percentile throughput is almost the same. This result shows how power control affects the network performance greatly. The used power control scheme sets a lower limit on the transmit power of the UEs than the one used in Setting 1; this causes the UL interference level in the network to be lower than the previous case which, in turn, allows the cell edge UEs to achieve a higher throughput when connected to a suboptimal cell in terms of pathloss.

On the other hand, the 50th percentile UEs do not achieve a higher throughput with the load balancing effect due to the lower bound on the UEs transmit power. These UEs hence might not be able to use all the resources available to them; therefore, these UEs achieve a relatively low gain from the higher resource availability whereas the lower link quality to the suboptimal cell reduces the throughput. Consequently, both effects almost even out and there is no gain in terms of 50th percentile throughput.

The main message in Figure 5.8 is that cell edge UEs are mostly interference limited whereas 50th percentile UEs are power limited so having power control Setting 1 would benefit the 50th percentile UEs but would be harmful for cell edge UEs while power control Setting 2 has the opposite effect.

Figure 5.9 shows a CDF of the variance of the UEs UL SINR over time for Setting 1 where interference is quite high. DUDe shows an average reduction of variance of about 10dB compared to DL-RSRP whereas DUDe-Load shows an even lower average variance of about 15dB compared to DL-RSRP. The lower variance reflects a more stable interference scenario in DUDe where the lower variance of DUDe-Load results from the improved load balancing effect which improves the resource utilization and, in turn, helps in stabilizing the interference. This is a very important feature since UL interference is known to be very volatile and dynamic and this result shows that radio resource management (RRM) and self-organizing network (SON) operation in general can be facilitated using DUDe.
Figure 5.9 CDF of the SINR variance where DUDe clearly shows interference calming properties.

Figure 5.10 5th, 50th and 90th percentile throughput of the three cases with interference aware power control.

In Figure 5.10, we show throughput results for the interference aware power control in (5.6). The aim here is to try to find a trade-off between 5th and 50th percentile performance. We see, indeed, that using this power control setup we achieve a similar or even higher 5th percentile throughput as in Setting 2 in Figure 5.8 where DUDe-Load outperforms DUDe by 15%. Also, in the 50th percentile the performance is similar to Setting 1 in Figure 3 where DUDe-Load outperforms DUDe by 20%. The better performance of DUDe-Load in the 5th and 50th percentile throughputs results from the fact that the interference aware power control affects more the UEs that cause higher interference, mostly cell edge UEs, to neighboring cells while allowing the other UEs, 50th and 90th percentile UEs, to transmit with a higher power. This results in a lower interference scenario which benefits the cell edge UEs that are interference limited and also allows the higher achieving UEs to transmit with a higher power and, in turn, exploit the extra resources resulting from load balancing.
In the results in Figure 5.11 we study the throughput behaviour in the 3 cases while changing the backhaul capacity of Scells from 1 to 100 Mbps. The Mcells backhaul capacity is assumed to be 100 Mbps in all cases. We present the results for the interference aware power control setup used in Figure 5.10.

![Figure 5.11 Throughput percentiles against backhaul capacity](image)

In the 5th percentile result the DUDe-load case shows the highest throughput since the UEs know of the backhaul and load capabilities of the cells. The DL-RSRP case performs better than the DUDe case up to a backhaul capacity of 10 Mbps after which DL-RSRP saturates and DUDe keeps on increasing.

Similarly, in the 50th percentile the DL-RSRP case is performing almost the same as DUDe-Load for very low Scell backhaul capacities since in the former case the UEs are mostly connected to the Mcells but as the Scell backhaul capacity increases DL-RSRP starts saturating and DUDe-Load surpasses it. Also the DUDe-load case is outperforming DUDe for the different capacities where the gain increases as the backhaul capacity of Scells increases as with the increase of Scell capacity DUDe-Load can have more options to assign UEs to Scells in a more efficient way.

Finally for the 90th percentile throughput, DUDe outperforms both DUDe-load and DL-RSRP since it has the lowest number of UEs connected to the Mcells. These UEs can get very high throughputs, up to a certain point where DL-RSRP surpasses DUDe. The reason is that Scells in DL-RSRP serve fewer UEs than the other 2 cases. Therefore after a certain backhaul capacity Scells can provide very high data rates to these UEs. Looking at the DUDe-load case, with lower Scell backhaul capacities the UEs are pushed more towards the Mcells but
still DUDe-load has less UEs connected to Mcells than DL-RSRP which explains why DUDe-load outperforms DL-RSRP at the beginning but as the Scells backhaul capacity increases the load balancing role is stronger which stops the 90th percentile throughput of DUDe-load from increasing as explained before.

Finally, in order to have some insight on the load balancing effect of DUDe we compare the variance of the number of UEs per cell in the 3 cases. This measure gives an indication of how UEs are distributed among the cells. A high variance indicates low load balancing effect and vice-versa. The variance is 470, 83 and 21 for DL-RSRP, DUDe and DUDe-load respectively. The DUDe case shows a clear improvement of load balancing over DL-RSRP which is shown by a dramatically reduced variance which, in turn, shows that the variation in the number of UEs/cell is small. The DUDe-load case shows an even lower variance (i.e. better load balancing) than DUDe as it is not only restricted on balancing the UEs between Mcells and Scells but it also improves the load balancing among Scells which is a very important feature in future ultra-dense Scell networks.

The decoupling of the downlink and uplink, referred to as DUDe, is an emerging paradigm shown to improve capacity significantly for cell edge users. The underlying principles of DUDe relate to a proper and independent association of the uplink and downlink. The focus of this paper has thus been to extend the prior simple association algorithms, based on the link quality in the respective links only, to a more advanced approach which considers the load in the cells as well as any backhauling constrains.

Having first introduced the general system architecture, the association algorithms as well as the simulation framework, we then presented and discussed an ample amount of performance results. The findings confirm that the enhanced DUDe achieves a reduced UL SINR variance over baseline LTE, in the order of 10-15 dB, which facilitates RRM and SON operations. Results for our load-aware DUDe show that the system throughput improves even further compared to the prior introduced baseline DUDe approach. The performance improvement depends very much on the power control mechanism used.

We have shown performance results for different power control settings and we used an interference aware power control algorithm where throughput gains of the load aware DUDe over baseline DUDe are 15% and 20% in the 5th and 50th percentile throughput respectively. We believe that the DUDe technique is a strong candidate for 5G architecture designs and it can be very useful in many applications like real-time video gaming, Machine Type Communications (MTC), among others, where uplink optimization is very critical. In the next
section we discuss some architecture considerations for 5G networks based on today’s LTE architecture that could facilitate the implementation of DUDe in future networks.

5.6  RAN Architecture considerations for DUDe

Decoupling the downlink from the uplink naturally requires some changes in the overall architecture, spanning from the radio access network (RAN) to the core network (CN). Decoupling flow notably requires some tight control to enable a smooth flow splitting and flow reassembly. From a physical channel perspective, two possible solutions have been proposed in [5.22] during the Rel. 12 works. Herein we study DUDe-enabling architectures from the perspective of Access-Stratum (AS) and Non-Access Stratum (NAS) signaling. AS signalling refers to Layer 1, Layer 2 and RRC control messages exchanged between UE and BS. NAS signaling refers to control messages exchanged between UE and core-network. It includes e.g. establishing and managing bearers, authentication and identification messages, mobility management and tracking area update [5.23]. We note that delay, reliability and throughput requirements for each of these are very different, which naturally leads to a plethora of possible architectures.

To this end, we propose a few options which rely on the following assumptions:

- a baseline 3GPP architecture;
- availability of Multi-Flow TCP, which is able to handle different data flows at networking layer;
- CoMP, which defines RAN traffic anchor points;
- support of multi-homing which allows different RATs to be connected at the same time to the same UE.

The options discussed below are mainly differentiated in the layer where the separation occurs, as well as the anchor point of choice where the traffic is reunited again. The different choices are depicted in Fig. 5.13 and discussed below.
**Figure 5.12. Architecture considerations**

- **NAS-Decoupling with RAN Anchor Point.** In this proposal a dedicated AS bidirectional connection is kept for both the Mcell and the Scell. The strength of this architecture is that all the delay-sensitive signaling (such as H-ARQ signaling) is handled by each cell. However, this requires the use of bidirectional physical control channels. Moreover, we note that having the traffic-merging anchor point in the RAN requires the traffic to be handled via an established X2 interface, similar to the CoMP procedures currently outlined in 3GPP. NAS signaling is also handled via the X2 interface.

- **NAS-Decoupling with CN Anchor Point.** This proposal differs from the previous one for the fact that data and NAS signaling are directly routed from the Scell to the core. The advantage here is that mobility is handled in a more efficient way, at the expense of delays due to the MME, which often resides physically far from the RAN. Also in this case, delay-sensitive signaling is handled via bidirectional exchanges at each cell.

- **AS-Decoupling with RAN Anchor Point.** In this embodiment, the most aggressive of all, there is a complete separation of the traffic; i.e. if the UE communicates in the UL to the Scell, no DL is maintained. This requires AS and NAS information to be sent with minimal delay via the DL of the Mcell. The disadvantage here is that the X2 needs to facilitate close-to-zero delay communications; the advantage is that radio capacity is completely freed in the decoupled link.
We note that we didn't consider the case of AS-Decoupling with CN Anchor Point, due to the fact that for delay-sensitive control signaling is not possible to tolerate delays due to the anchor in the CN often physically residing far from the RAN.
6. Underlay Device-to-Device Communications for LTE-A Wireless Networks

6.1 Introduction

Device-to-device (D2D) communication enables nearby mobile devices to establish direct links in cellular networks, unlike traditional cellular communication where all traffic is routed via base stations (BSs). D2D has the potential to improve spectrum utilization, shorten packet delay, and reduce energy consumption, while enabling new peer-to-peer and location-based applications and services and being a required feature in public safety networks. Introducing D2D poses many challenges and risks to the existing cellular architecture. In particular, in a D2D underlaid cellular network where the spectrum is reused D2D transmission may cause interference to cellular transmission and vice versa. Existing operator services may be severely affected if the newly introduced D2D interference is not appropriately controlled. In this section, we focus our attention on the application of network coding (NC) to cooperative wireless networks and device-to-device (D2D) communications.

In particular, we have carried a set of analytical studies with numerical simulations to find a transmission scheme for reliably conveying information in cooperative networks using NC [6.1]. The key idea behind these studies is to study and analyze a set of Random Linear Network Coding (RLNC) [6.2] based schemes in the State-Of-The-Art (SoA), that allows a mobile network, for example LTE-A, to decrease the number of transmissions for a set of devices sharing common information content even in the presence of high packet losses. The reason for this choice is two fold: (i) RLNC based schemes have proven to provide significant gains for single hop and wireless mesh heterogeneous topologies [6.4] and (ii) for cooperation based RLNC schemes, involved devices typically will need less transmissions from the system or among themselves compared to other classical strategies [6.3]. This kind of approach reduces the network load in terms of total transmitted power meaning a reduction in energy consumption and hence, the overall interference. Also, by reducing the amount of transmissions, network throughput is increased due to the completion time for the same information content sharing also being reduced [4-7].

For our analytical studies, we review different transmission scenarios and schemes in typical mobile network topologies. The key point is to evaluate which are the benefits compared to SoA network coding schemes and when are they preferable than other conventional techniques under a set of performance metrics for the given scenarios [6.10]. Also, a new scheme for low overhead in cooperative networks is currently being proposed as part of these studies [6.11]. First, we provide an introductory motivation and definition for the
general problem of reliably transmitting information to a set of devices in a wireless network with intra-session based NC cooperation mechanisms. Then, we provide a review of RLNC based techniques to observe the behaviour of the proposed schemes in terms of number of transmissions and metrics given scenarios.

6.2 RLNC transmission scenarios and schemes analysis

In this section we review and provide a set of available techniques that may be applied for our scenario of interest. To this end, we first define the problem of reliably transmitting a common information content among a set of receivers. Then, we review different transmission schemes in order to identify which are suitable strategies according to a set of metrics.

6.2.1 Network and System Model

We consider the problem of reliably transmitting a batch of $g$ packets from a source to $N$ receivers in a single hop network as shown in Fig. 6.1. Each packet has the same length of $B$ bits. We assume a fixed time slotted system to which all nodes are synchronized which is a suitable case for LTE-A. We consider independent heterogeneous packet erasure rates on the connectivity links from the source to the receivers, $\epsilon_j, j \in [1,N]$, e.g. the packet reception distribution of receiver $j$ is $Bernoulli(1 - \epsilon_j)$ and is independent from all others in general.

![Figure 6.1. Single source - multiple sink topology of $N$ receivers with erasures](image)

Besides the single hop connections, receivers might be in a fully interconnected fashion which we call a mobile cloud [6.12]. Hence, the connections can be regarded as bidirectional symmetric channels in a local area technology with a homogeneous erasure rate $\epsilon$. We also assume that in a time slot a device is either transmitting, receiving or waiting for reception and that a successful transmission is received in the same slot it was sent. Lastly, we consider that an ideal error free instantaneous orthogonal feedback channel exists in order to acknowledge packet reception when required.
We review two transmission scenarios, e.g. broadcast with RLNC and cloud cooperation with RLNC since these schemes outperform their uncoded counterparts as described in [3, 13-14]. We will give a brief description of them to indicate the behaviour of the encoding, decoding and (if necessary) recoding process. In our work, we differentiate our schemes according the Galois fields choices in a given generation. We separate them by (i) the single field schemes, in which the same Galois field $q$ is used for all coding coefficients and (ii) the multiple fields schemes where a Galois field $q_i, i \in [1,g]$, is picked for each coding coefficient $c_i, i \in [1,g]$ in the generation. We will specify the fields choice and finite fields arithmetics and operations for the multiple fields schemes when we address them in the next sections.

We provide an in-depth study of the distributions for the number of transmissions for each of the given scenarios and schemes. We separate the analysis on a field scheme basis since the computation of the basic probability mass function (pmf) heavily depends on it. In this way, we model the number of transmission as random variables using the geometric distribution as a building block to derive the pmf in order to obtain a complete description of the transmission process. We first give a new expression of the probability mass function for RLNC with no erasures and then compute the pmf for the transmission scenarios and schemes.

6.2.2 Transmission Scenarios

Broadcast with RLNC

For broadcast, the source generates encoded packets from a generation of size $g$, where each packet has $B$ bits. For the encoding process, coded packets are typically generated by selecting random coefficients from some particular choice of Galois fields of size $q$, namely $GF(q_i)$ for each coding coefficient in general, in order to include both single and multiple field schemes. Then, the source attaches the coding coefficients values in bits as overhead required for the decoding process. Following, the source broadcasts each coded packet to all receivers through the packet erasure channel described in the previous section. When a packet successfully arrives at a receiver, it checks if the packet is linearly independent from all previous. If not, it discards it and waits for a new one. In case of being linearly independent, the receivers adds it to the coding matrix in order to efficiently perform Gaussian elimination for decoding. In this way, an acknowledgement is sent when all receivers have collected their required combinations through an ideal feedback channel of the model. If this is not the case, the source keeps sending coded packets until this occurs. For this scenario, recoding is unnecessary.
Cloud Cooperation with RLNC

In a cloud cooperative scenario, packet transmissions take place in two stages: (i) between source and receivers as group and (ii) internally in the cloud between receivers which have missing packets, which we label the remote and local stage respectively. For the remote stage, the source creates a telescopic coded packet, as described above, and broadcasts it for the cloud to get it collectively, meaning that it is enough that at least one receiver gets a coded packet to consider the cloud has it. This stage finishes once the cloud gets the generation as a group. After the remote stage has ended, there will be receivers which do not have \( g \) linearly independent equations to decode. However, to manage this situation, in the local stage each receiver broadcasts recoded packets from the same field in the single field schemes. For the multiple fields scheme, a packet is recoded with all the coefficients of the recoding vector picked from lowest field in the generation, e.g. \( \min(q) \). This recoding method provides a simple way to recode but has an impact in the decoding probability \[6.11\].

Broadcasting in this stage occurs in a sequential fashion in order to guarantee that each receiver distributes all its knowledge to the set. At the end, the local stage finishes once all receivers have decoded the generation and any node sends an acknowledge to the source.

6.2.3 Distribution Preliminaries

We provide some definitions for the geometric distribution given that it will be a basic tool for our derivations. We focus on this distribution given that its a reasonable standard model for packet distributions and allows us to easily compute more complicated distributions for other scenarios and schemes.

To avoid ambiguity, we employ the definition of the geometric distribution that stands for the number of Bernoulli trials (each of success probability \( p \)) required to get one successful event which is defined as follows:

\[
T \sim Gcom(p) \iff Pr[T = t] = (1 - p)^{t-1}p , \ t = 1, 2, \ldots
\]  

(6.1)

From this distribution, we will often use its moments which can be obtained from (6.1) and defined as follows:

\[
E\{T\} = \frac{1}{p} , \ Var\{T\} = \frac{1 - p}{p^2}
\]  

(6.2)

Another relevant property of the geometric distribution is its probability generating function (pgf). This property allows us to represent any distribution as a function \( G(z) \), in a complex \( z \) domain, retaining all its properties. The pgf for the geometric distribution is defined as:
The pgf in (6.3) is defined in the complex domain with a region of convergence for which the pgf series converges. Finally, we notice that the pgf transformation over the original pmf holds a relation with the Z-transform of a discrete sequence. Let \( P(z) \) be the Z-transform of a pmf. Then, the relationship between both transformations is defined in equation (6.4). We see from it that the relationship is just evaluate analytically in the inverse of the argument. This relationship will play a role later when deriving the pmfs for our scenarios and schemes.

\[
    P_1(z) = \mathcal{Z}\{ Pr[\mathbb{T} = t] \}(z) = \sum_{t=1}^{\infty} P_r[\mathbb{T} = t]z^{-t} = G_T(z^{-1})
\]

\[ (6.4) \]

### 6.2.4 Single Field Schemes

**RLNC Probability Mass Function Without Erasures**

For the single field cases, all the coding coefficients are drawn uniformly at random from \( GF(q) \). As already mentioned, we first calculate the pmf for the number of transmissions required for decoding for a single link using RLNC with no erasures as shown in Fig. 6.2.

![Source - Sink RLNC link](image)

**Figure 6.2. Source - Sink RLNC link**

Although, previous work exists in this area [15-16], the previous definitions will be helpful for obtaining other distributions for which the current analysis tools becomes cumbersome. A model for the required number of transmissions for decoding RLNC packets employs an Absorbing Markov Chain as shown in Fig. 6.3.

![Absorbing Markov Chain for RLNC](image)

**Figure 6.3. Absorbing Markov Chain for RLNC**
This chain comprises $g+1$ states. First, state $i$, with $i \in [1,g]$, is the case where the $i$-th linearly independent (l.i.) coded packet has not been received by the destination. Then, $D$ is the absorbing state where decoding is performed. The transition probabilities of each state depend only on the amount of previously received l.i. combinations. Using the tools from section 6.2.3, we model the amount of transmissions to leave any stage of the chain in Fig. 6.3 as a geometric distribution with a success probability depending on the state as follows:

$$T_{RLNC,i} \sim Geom(p_i), \quad p_i = 1 - q^{-g+i-1}, \quad i \in [1,g] \quad (6.5)$$

With the previous definition, the total number of transmissions to decode is just the sum of each of the random variables in (6.5) as follows:

$$T_{RLNC} = \sum_{i=1}^{g} T_{RLNC,i} \quad (6.6)$$

However, analytically computing the pmf for the sum of several random variables becomes very demanding since it involves $g \cdot 1$ discrete convolutions. Instead, we use a pgf approach to get the analytical pmf. To achieve this, we use (6.3) to get the pgf of each of the stages and use the fact the pgf for the sum of independent random variables is the product of their pgfs. Also, we use the linear dependence probabilities, defined as $\gamma_i = 1 - p_i, i \in [1,g]$, in order to simplify our calculus. Then, the pgf becomes as follows:

$$G_{T_{RLNC}}(z) = \prod_{i=1}^{g} G_{T_{RLNC,i}}(z)$$

$$= \prod_{i=1}^{g} \frac{p_i z}{1 - \gamma_i z}$$

$$= P_g z^{g-1} \prod_{i=1}^{g} \frac{1}{1 - \gamma_i z}, \quad |z| < \min(\gamma_i^{-1}) \quad (6.7)$$

In (6.7), for the third equality, $P_g$ stands for the product of all the linearly independent probabilities from (6.5). We recognize this scalar term as the probability of decoding in exactly in $g$ transmissions. We use the relationship described in (6.4) to further simplify our computations. In this way, and rearranging terms, the Z-transform of the RLNC pmf without erasures is as follows:

$$P_{T_{RLNC}}(z) = G_{T_{RLNC}}(z^{-1})$$

$$= P_g z^{-g} \prod_{i=1}^{g} \frac{1}{1 - \gamma_i z^{-1}}, \quad |z| > \max(\gamma_i) \quad (6.8)$$

With the Z-transform of the pmf in (6.8), we perform a partial fraction expansion since the product term of the second equality is rational. This turns into the following:
\[ P_{\text{RLNC}}(z) = P_g z^{-g} \sum_{i=1}^{g} \frac{a_i}{1 - \gamma_i z^{-1}}, \ |z| > \max(\gamma_i) \]  
(6.9)

In (6.9), we have split the product as a sum of \( g \) terms since all the poles of the Z-transform in (6.8) are simple due to the fact the linearly dependent probabilities are unique. The \( a_i \) term in the summation is the residue for the \( i \)-th term which is easy to calculate as follows.

\[ a_i = \prod_{m=1, m \neq i}^{g} \frac{1}{1 - q_{m-i}}, \ i \in [1, g] \]  
(6.10)

Afterwards, with the residues values, we use simple inverse z-transforms pairs to obtain the pmf for RLNC without erasures, as follows:

\[ f_{\text{RLNC}}(t; g, g) = P[T_{\text{RLNC}} = t] = P_g \sum_{i=1}^{g} a_i \gamma_i^{t-g}, \ t \in [g, \infty) \]  
(6.11)

**Unicast RLNC with Erasures**

Given that we have computed the RLNC pmf in (6.11), for a unicast pmf with erasures we now account for the erasure process for a single link as described in Fig. 6.4.

![Figure 6.4. Source - Sink Unicast (RLNC) link](image)

For a unicast session, we need to have \( g \) linearly independent received packets in \( t \) transmissions. Therefore, we need to consider all the cases where \( i \) packets are received (with the final success in \( t \), which [6.15-6.16] do not consider) and \( t-i \) packets were lost or linearly dependent. For this, we review two main probabilities in the same way as described in [6.17,6.10]. First, the probability for successfully receiving \( i \) linearly independent coded packets in \( t \) transmissions by just considering erasures, is \( NB(i, 1 - \epsilon) \) which is the negative binomial probability for \( i \) trials and success probability \( 1 - \epsilon \), defined as follows:

\[ Pr[S_t = t] = \binom{t - 1}{i - 1} (1 - \epsilon)^i \epsilon^{t-i} \]  
(6.12)

Second, the probability that \( g \) coded packets are linearly independent in \( i \) slots, is just (11). Then by the total law of probability, the probability of decoding in exactly \( t \) slots for a single user with RLNC based unicast with erasures is:
Broadcast RLNC with Erasures

The same broadcast RLNC scenario as above is considered where all the coding coefficients are generated from the same field. In this situation, each receiver needs to collect different linear combinations to decode the packets. Then, we can regard the distribution of this scenario as the case of finding the distribution for the maximum of $N$ unicast sessions. In this manner, the number of transmissions will be determined by the receiver that performs the worst in terms of retransmissions, the random variable for the number of transmission in a broadcast scenario is modelled as:

$$T_B = \max_{j=1,\ldots,N} T_{Uj}$$  \hspace{1cm} (6.14)

We calculate (6.14) by using a cumulative distribution function (cdf) approach. For the probability of the maximum being less than or equal to $t$ transmissions, this must occur for all receivers. Next, under the independence assumption, we compute the cdf for broadcast RLNC as shown as follows:

$$F_{T_B}(t; N, \epsilon_1, \ldots, \epsilon_N, g, q) = Pr[T_B \leq t] = \prod_{j=1}^{N} Pr[T_U \leq t] = \prod_{j=1}^{N} \left( \sum_{i=g}^{t} \sum_{k=g}^{k} (k - 1) \epsilon_j^k \epsilon_j^{k-1} f_{T_{RLNC}}(i; g, q), \ t \in [g, \infty) \right)$$  \hspace{1cm} (6.15)

With the cdf in (6.15), the pmf from broadcast can be obtained from the cdf as follows:

$$f_{T_B}(t; N, \epsilon_1, \ldots, \epsilon_N, g, q) = Pr[T_B = t] = Pr[T_B \leq t] - Pr[T_B \leq t - 1] = F_{T_B}(t; N, \epsilon_1, \ldots, \epsilon_N, g, q) - F_{T_B}(t - 1; N, \epsilon_1, \ldots, \epsilon_N, g, q), \ t \in [g, \infty)$$  \hspace{1cm} (6.16)

With (6.16), we compute all the values of the pmf. For the particular case of $t = g$, the subtracted term in the third equality is zero by definition.

Cloud Cooperation RLNC with Erasures

We conduct the analysis considering the possibility of users being only connected to the remote mobile network and users being connected to the local network to observe the effect in the performance metrics that we will be detailed in the next sections. We refer to the users
connected to both the mobile network and the cloud as *heads* and the ones connected only to the cloud as *non-heads*. In general, we consider that there are $H$ heads, with $H < N$.

To compute the pmf for this case, we separate the pmf in two parts, one for each stage. For the remote stage, given that all links need to fail for a packet to not be received, we can think of the pmf of the remote stage as the case of a unicast pmf from (6.13) but with a lower equivalent erasure probability as follows:

$$\mathbb{T}_{CC,r}(H, \epsilon_1, \ldots, \epsilon_N, g, q) = \mathbb{T}_U \left( \prod_{j=1}^{H} \epsilon_j, g, q \right)$$

(6.17)

The parenthesis notation in (6.17) indicates that we evaluate the pmf in (6.13) with the shown parameters. In [6.9], for the pmf in this stage we excluded the transmissions among the heads because: (i) the following broadcast stage becomes the dominant factor in the overall pmf in general and (ii) for a high number of heads, the pmf will be highly concentrated at $g$ since all packets are received by at least one head.

Similarly, we regard the pmf of the local stage as the case of a broadcast pmf with the same parameters because the non-heads have not received any packets from the source. Besides, given that all heads broadcast their combinations in a sequential fashion, at the end they behave as an equivalent source for the non-heads. Thus, the pmf for the local stage is:

$$\mathbb{T}_{CC,l}(N-H, \epsilon, g, q) = \mathbb{T}_B(N-H, \epsilon, g, q)$$

(6.18)

Then, the pmf for this scenario is the pmf of the sum of the random variables in (6.17) and (6.18). When we compute the pmf with this operation, we properly shift the pmf for the local stage to the origin since the pmf of local stage part in (6.17) already takes into account that at least $g$ transmissions are necessary. Finally, to ensure that the broadcast part is always included to dominate the pmf for cloud cooperation, we will evaluate it in the range $N > H$ in the next sections.

### 6.2.5 Multiple Fields Schemes

For this scheme, each coding coefficient $c_i$ is uniformly and randomly chosen from $GF(q_i)$ with $q_i = 2^k_i, k_i \in \mathbb{Z}^+$. The schemes described in this section are based on telescopic codes [6.11], which in turn are based on the principle of RLNC, but some differences exist. The key idea behind telescopic codes is that they rely on composite extension fields. An extension field is constructed from a lower base field where operations are simpler. The sum and product operations in the extended field are made of functions in the base field. These operations are designed in order to keep compatibility between common elements in both the extended and original field. The end goal of composite fields is to allow different finite field
arithmetic in the same generation. The main purpose of employing telescopic codes is to keep a balance between the benefits of low decoding probability and low coding coefficients overhead. The feature of selecting elements from different fields in the generation impact our models given that the basic pmf of a single link without erasures changes.

**Telescopic Codes Probability Mass Function without Erasures**

As in the RLNC case, we first calculate the pmf for the number of transmissions in a single link using telescopic codes. As mentioned in the previous sections, the main difference between these type of codes and RLNC is the use of different field sizes in the same generation. This difference affects the pmf for these schemes since the absorbing Markov chain used to represent the decoding process for RLNC needs to be modified in order to include the effect of the multiple fields. In [6.11], it is shown that the presence of multiple fields generates different pivot types (depending on the field size) in the coding matrix. The reason behind this is that each pivot is associated with a coefficient position. However, in [6.11] it is shown that under some general assumptions, the decoding process is dominated by the pivot of the lower field that has not been received yet. Thus, a reasonable Markov Chain model is shown in Fig. 6.5.

![Figure 6.5. Absorbing Markov Chain for Telescopic Codes](image)

In principle, the Markov Chain in Fig. 6.5 has the same amount of states, with their interpretation, than the one for RLNC in Fig. 6.2. Now, however, the transition probabilities of each state depend not only on an innovative packet reception, but also on the field size for that state. This is related to the search of the considered pivot for that state. Without loss of generality, in the previous model, we consider \( q_i \leq q_{i+1}, \forall i \in [1, g - 1] \).

We use a procedure similar to that of previous sections. The major difference of this procedure is in the calculation of the inverse Z-transform of the pmf for telescopic codes. For this type of scheme, equation (6.8) needs to be modified as as follows:

\[
P_{T_{telescopic}}(z) = P_g z^{-g} \prod_{i=1}^{L} \frac{1}{(1 - \gamma_i z^{-1}) m_i} , \quad |z| > \max(\gamma_i)
\]

(6.19)
For the Z-transform of the pmf, we observe that the presence of multiple fields in the Markov Chain of Fig. 6.5 makes the transition probabilities of some states equal to others in general. This introduces poles of different order in the pgf. So, we consider \( L \leq g \) distinct \( \gamma_l, l \in [1, L] \) probabilities in the Markov Chain of Fig. 6.5. Each \( \gamma_l \) has an associated multiplicity \( m_l \) which is the number of times it repeats in the chain and also the order of the \( \gamma_l \) pole in the Z-transform of the pmf. Now the partial fraction expansion changes as shown as follows.

\[
P_{TC}(z) = P_g z^{-g} \sum_{l=1}^{L} \sum_{n=1}^{m_l} \frac{a_{l,n}}{(1 - \gamma_l z^{-1})^n} ; \quad |z| > \max(\gamma_l)
\]

\[
a_{l,n} = \lim_{z \to \gamma_l} \frac{d^{m_l-n} \left[ \frac{G_{TC}(z^{-1})}{P_g z^{-g}} \right]}{d(z^{-1})^{m_l-n}} \frac{(1 - \gamma_l z^{-1})^{m_l}}{(m_l - n)!} \quad \gamma_l \gamma_l, \quad l \in [1, L], \; n \in [1, m_l]
\]

Finally, based on previous results, the pmf of telescopic codes with no erasures can be formulated as follows

\[
f_{TC}(t; g, q) = \Pr[T_{TC} = t] = P_g \sum_{l=1}^{L} \sum_{n=1}^{m_l} a_{l,n} \left( \frac{t - g + n - 1}{t - g} \right) \gamma_l^{t-g}, \; t \in [g, \infty)
\]

**Broadcast with Telescopic Codes**

To obtain the pmf for broadcast with telescopic codes, given that we separate the coding scheme from the erasure process, we need to use the pmf of telescopic codes instead of that of RLNC in (6.15). By doing so, the cdf for broadcast with telescopic codes.

\[
F_{B,TC}(t; N, \epsilon_1, \ldots, \epsilon_N, g, q) = \Pr[T_{B,TC} \leq t]
\]

\[
= \prod_{j=1}^{N} \left( \sum_{k=g}^{t} \sum_{i=g}^{k} \binom{k-1}{i-1} (1 - \epsilon_j)^i \epsilon_j^{k-i} f_{TC}(i; g, q) \right), \; t \in [g, \infty)
\]

As a consequence, the pmf using broadcast can be obtained from the cdf in (6.23) as follows:

\[
f_{B,TC}(t; N, \epsilon_1, \ldots, \epsilon_N, g, q) = \Pr[T_{B,TC} = t]
\]

\[
= \Pr[T_{B,TC} \leq t] - \Pr[T_{B,TC} \leq t - 1]
\]

\[
= F_{B,TC}(t; N, \epsilon_1, \ldots, \epsilon_N, g, q) - F_{B,TC}(t - 1; N, \epsilon_1, \ldots, \epsilon_N, g, q), \; t \in [g, \infty)
\]

Again, with (6.24), we can compute all the values for the pmf of Broadcast with Telescopic Codes. For the particular case of \( t = g \), the subtracted term in the third equality is zero by definition.
Cloud Cooperation with Telescopic Codes

To compute the pmf for this case, we separate the pmf in two parts, one for the remote stage and one for the local stage. For the first stage, we proceed as in previous sections and get:

\[ \mathbb{T}_{TC,CC}(N, \epsilon_1, \ldots, \epsilon_N, g, q) = \mathbb{T}_{TC,U} \left( \prod_{j=1}^{N} \epsilon_j, g, q \right) \]  

(6.25)

The parenthesis notation in (6.25) indicates that we evaluate the pmf in (6.25) with the shown parameters. Now, for this case, the local stage becomes an update stage for the devices that still have remaining linearly independent packets to collect. In this manner, we regard the pmf of the local stage as a broadcast pmf evaluated with the new parameters that we calculate.

First, we need to consider the number of devices that have enough linearly independent packets to exclude them from the broadcast phase. After \( g \) transmissions, for the considered homogeneous erasure rate scenario, any device has all required linear combinations with probability \( P_g(1 - \epsilon)^g \) since all need to be linearly independent and correctly received. So, on average, at least \( \text{floor}(NP_g(1 - \epsilon)^g) \) devices have all combinations. Second, the remaining receivers have \( \text{floor}(g(1 - \epsilon)) \) packets on average, so we need to transmit the difference which is \( \text{ceil}(ge) \). Third, for this stage, packets are recoded using the lowest field in the generation. Hence, with the new parameters, the pmf for the local of cloud cooperation with telescopic codes is defined in (6.26). At the end, the pmf for this scenario is the pmf of the sum of the random variables in (6.25) and as follows:

\[ \mathbb{T}_{TC,CC}(N, \epsilon, g, q) = \mathbb{T}_{TC,B}(N - \lceil NP_g(1 - \epsilon)^g \rceil, \epsilon, \lceil ge \rceil, \min(q)) \]  

(6.26)

6.2.6 Performance Metrics

With the distributions for the number of transmissions for the different schemes from sections 6.2.4 and 6.2.5, we proceeded to make a comprehensive analysis for the previously mentioned transmission strategies in terms of mean and variance of the pmfs. Then, a set of derived metrics need to be reviewed for analysis.

For the single field schemes, user average energy consumption is evaluated from an energy perspective. Also, the stable throughput, which we define in the following section, is studied for each transmission scenario. In the cloud cooperation scenario, we change the number of users with cellular connections. In this way, the cooperation analysis is made for a number of users higher than the number of heads. Moreover, our study is focused on comparing broadcast and cooperative scenarios.
For the multiple fields schemes, we study the mean overhead since we want to observe the behaviour of different schemes in terms of the extra bits introduced by the linear dependence when including low fields and by the coding coefficients when including high fields. By taking into account the overhead, we make a search for the optimal field scheme that minimizes the overhead for a given scenario. Therefore, for each scenario, we review the overhead for three multiple field schemes, namely, all coding coefficients in $GF(2)$, all coding coefficients in $GF(2^8)$ and the optimal field scheme of the given scenario, as previously defined.

**Single Field Metrics**

**Throughput:** The throughput for broadcast RLNC for a given number of users, erasure rate, generation and field size is defined as follows:

$$R_B = \frac{g}{T_{\text{cel}B}(N, \epsilon, g, q)} \text{[packets/s]}$$  \hspace{1cm} (6.27)

The throughput in (6.27) is evaluated using the results from previous sections. $T_{\text{cel}}$ is the time slot duration for the cellular stage. For the cloud cooperation scenario, we define the stable throughput as follows

$$R_{CC} = \frac{g}{\max(T_{\text{cel}CC}(H, \epsilon, g, q), T_{\text{loc}CC}(N - H, \epsilon, g, q))} \text{[packets/s]}$$  \hspace{1cm} (6.28)

where $T_{\text{loc}}$ stands for the time slot duration in the local stage.

With the previous definitions, we calculate the mean values to get average throughput in both cases. We performed a numerical analysis with the following values: $1 \leq N \leq 50, g = \{64, 128\}, q = 2^8, 0 \leq \epsilon \leq 0.6$. The time slot duration is set to $T_{\text{cel}} = T_{\text{loc}} = 0.5 \text{ ms}$ to be compliant with the LTE-A E-UTRA [6.18] and, in particular, for the specifications decided for D2D applications. Representative results are summarized in Fig. 6.6.

![Figure 6.6. Throughput for Broadcast and Cloud Cooperation based RLNC. Cloud Cooperation is evaluated with different heads. $g = 64, q = 2^8, \epsilon = 0.4$ [6.10]](image-url)
Figure 6.6 shows the throughput as defined in (6.27) and (6.28) for different scenarios. For all the scenarios, increasing the number of users reduces the throughput since we need to account for more transmissions. The highest throughput is obtained when we have an amount of users equal to the number of heads, as this reduces the work in the local phase. For a fixed amount of heads, increasing the number of receivers makes the throughput for cooperation to approach that of broadcast. This implies that broadcast transmission becomes the dominant effect.

**Energy:** Also, we evaluated the average energy consumption per device for the analyzed scenarios. We define the energy for each device as follows:

\[ E_{R_{\text{cell}},B} = E_{\text{cell}} T_B(N, \epsilon, g, q) \text{ [Joules]} \] (6.29)

\[ E_{R_{\text{cell}},CC} = E_{\text{cell}} \left( \frac{H}{N} \right) T_{\text{cell}}(H, \epsilon^H, g, q) + E_{\text{cell}} T_{\text{cell}}(N - H, \epsilon, g, q) \text{ [Joules]} \] (6.30)

Similar to the throughput durations, in (6.29) and (6.30), \( E_{\text{cell}} \) and \( E_{\text{loc}} \) are the energy costs per packet sent in the cellular and local stages, respectively. These values are a function of the energy spent per transmitted bit and also of the number of bytes per packet. However, since the last one is just a scaling factor and does not affect our analysis, we arbitrarily set it to 500 bytes which is standard size for network transmissions, although other values can be considered. The energy per bit values are extracted from [6.19]. We evaluate (6.29) and (6.30) and take the average value, to get the mean energy consumption of the devices. The results are summarized in Fig. 6.7.

![Figure 6.7. Energy spent per device for Broadcast and Cloud Cooperation based RLNC.](image)

Cloud Cooperation is evaluated with different heads. \( g = 64 \), \( q = 2^8 \), \( \epsilon = 0.4 \) [6.10]
In Fig. 6.7, cloud cooperation performs worse since we need several transmissions in both stages. However, for a high amount of devices, the heads can be also incremented and the number of transmissions in the remote stages becomes \( g \) while for the local stage increases. This makes both performances similar in the high user regime.

**Cellular vs. Cloud links:** In the previous sections, we assumed the same data rates in both the cellular and the local stages. The same assumption was made for the energy costs. Now, we study how the previous performance trends change as a function of the rates and of the costs. These results are shown in Figure 6.8 and 6.9.

**Figure 6.8.** Throughput vs. Rate Ratio. \( N = 50, g = 64, q = 2^8, \epsilon = 0.4 \) [6.10]

**Figure 6.9.** Device Energy vs. Energy Ratio. \( N = 50, g = 64, q = 2^8, \epsilon = 0.4 \) [6.10]
Figure 6.8 shows how the throughput varies depending on the ratio of the cellular and local data rate, where the ratio is defined as shown in the figure. The ratios are obtained by fixing the cellular network and varying the local network. When the local network is slower than the remote one, cloud cooperation performs worse than broadcast. Conversely, for a faster local network, cloud cooperation performs the best. When the number of heads decreases, the throughput also diminishes because there are fewer heads each with an independent chance of receiving the packet. The throughput is the highest when the rates in the local network are two times those of the cellular network.

Figure 6.9 shows how the energy for different scenarios changes as a function of the energy per bit. For a higher energy cost in the local network than in the cellular network, cloud cooperation demands more energy than broadcast, due to the extra transmissions in the local stage. On the contrary, when the cost of the local links is less than the cost of the cellular links, cloud cooperation employs less energy than broadcast.

### 6.2.7 Comparison Between Cooperative and Non-Cooperative Schemes

In this section we study the erasure rate regions where cooperation performs better than broadcast for a given data rate or energy ratio. For this study, we first separate the erasure rate effect of cellular and local stages as follows:

\[
G_t = \frac{E\{T_B(N, \epsilon_{cel}, g, q)\}}{\max(r_i E\{T_{CC,i}(H, \epsilon_{cel}, g, q)\}, E\{T_{CC,l}(N - H, \epsilon_{loc}, g, q)\})} \tag{6.31}
\]

\[
G_e = 1 - \frac{r_{e} E\{H_N\} E\{T_{CC,i}(H, \epsilon_{cel}, g, q)\} + E\{T_{CC,l}(N - H, \epsilon_{loc}, g, q)\}}{r_{e} E\{T_B(N, \epsilon_{cel}, g, q)\}} \tag{6.32}
\]

We define the throughput and energy gains of cloud cooperation against broadcast, \(G_t\) and \(G_e\), as shown in (6.31) and (6.32) respectively. The gains are function of the ratios defined in Figs. 6.8 and 6.9. Selected numerical illustrations are provided in Figs. 6.10 and 6.11.

Fig. 6.10 shows the regions where cooperation provides a throughput gain, as defined in (6.31), for different erasure rates for cellular and local links. The contour lines show where broadcast and cooperation perform the same. In the region below each line, cooperation provides higher throughput than broadcast. Above the line broadcast performs better. In the case of a fast local network, \(r_i = 0.5\), cooperation almost always provides a gain compared to broadcast, even in cases where the local losses are much higher than the cellular ones.
Figure 6.10. For different data rate ratio values ($r_t$), the lines indicate where cooperation and broadcast provide the same throughput for various erasure rates on the cellular and local links. Below the each line, cooperation performs better for the respective ratio. $N = 50$, $H = 40$, $g = 128$, $q = 2^8$ [6.10]

Figure 6.11. For different energy cost ratio values ($r_e$), the lines indicate where cooperation and broadcast provide the same energy. Below the each line, cooperation performs better for the respective ratio. $N = 50$, $H = 40$, $g = 128$, $q = 2^8$ [6.10]

Analogously for the energy, Fig. 6.11 shows the regions where cooperation provides an energy gain, as defined in (6.32), for various erasure rates in the cellular and local links. The contour lines show where both scenarios perform the same. Below each contour line, cooperation demands less energy per bit than broadcast. Above the line, the opposite holds.
**Overhead**: We define the overhead for a field scheme $s$ and transmission scenario $t$ as:

$$\mathbb{O}_{s,t} = (B + |v|_{s,t})(T_{s,t} - T_{smin,t}) + |v|_{s,t}T_{smin,t} \text{ [bits]}$$  \hspace{1cm} (6.33)

In (6.33), the overhead is defined as a random variable that depends on the scheme and transmission scenario considered. $|v|_{s,t}$ is coding coefficients overhead in bits for the given scheme and scenario which are attached to each coded packet defined as follows:

$$|v| = \sum_{i=1}^{q} \left\lfloor \log_2(q_i) \right\rfloor \text{ [bits]}$$ \hspace{1cm} (6.34)

The coding coefficients in (6.34) depend only on the considered scheme.

In (6.33), $T_{s,t}$ is the number of transmissions for the given scheme and scenario and $T_{smin,t}$ is a random variable for the minimum amount of transmissions that the scheme might take in the given scenario. A reasonable approximation for this variable is the constant value $g$ since it is the lowest amount of transmissions that any scheme might take. Considering this and rearranging terms, (6.33) becomes:

$$\mathbb{O}_{s,t} = (B + |v|_{s,t})T_{s,t} - gB \text{ [bits]}$$ \hspace{1cm} (6.35)

Given that we are evaluating the overhead, a reasonable choice for the cost function is the mean overhead. Then, the optimal field scheme for a given scenario is the one that minimizes the following cost function:

$$\min_{q} \left( B + |v|_{q,t} \right) \mathbb{E}\{T_{q,t}\}$$

$$\text{s.t. } q_i = 2^{2k_i}, \ i \in [1, g], \ k_i \in \mathbb{Z}^+$$ \hspace{1cm} (6.36)

In the nonlinear integer problem defined in (6.36), we have substituted the scheme subscript for the fields list, $q$, to highlight the dependence on the fields for the mean overhead minimization.

### 6.3 RLNC Models for the ns-3 Simulator and the Steinwurf Kodo Library

Finally, we mention that the proposed protocols and solutions have been implemented in the ns-3 simulator by using the Kodo library that provides network coding functionalities. The development includes creating basic topologies and examples for using Kodo in ns-3. The developed material is available online as described in what follows:
• A tutorial designed to allow newcomers to perform ns-3 simulations with Kodo. The tutorial description and scope for further references can be found here: http://kodo-ns3-examples.readthedocs.org/en/latest/
• The project description, news and source code for further references can be found here: https://github.com/steinwurf/kodo-ns3-examples

6.4 Conclusions
We have presented an in-depth study of the specific operating regions where cloud cooperation delivers gains in throughput and energy with respect to broadcast transmission for single field schemes. Our numerical results showed that gains can be achieved even if long-range and short-range technologies transmit at comparable data rates. More importantly, we showed that cooperation can provide several fold gains in the best broadcasting option (network coded broadcast) as long as the short-range link is at least two times faster than the long-range link. Finally, our results showed that a moderate number of heads (e.g., three or more) per cooperative cluster is enough to yield the high throughput gains while maintaining a low energy consumption at the receivers.
7. References


